# Test (SL) – Probability Distributions

Syllabus content: ▪ discrete random variables and their probability distributions ▪ expected value (mean), E($X$), for discrete data ▪ binomial distribution ▪ normal distribution

total marks on test: **55**

Calculator allowed on all questions

**1.** Callum always practices a set of three penalty kicks at the end of each football training session. The discrete random variable $X$ represents the number of penalty kicks that Callum makes each session. The table below shows the probability distribution.

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $\mathrm{P}(X = x)$ | 0.05 | $k$ | 0.5 | 0.3 |

(a) Find the value of $k$. [2 marks]

(b) Find the expected number of penalty kicks Callum makes at a training session. [2 marks]

**2.** The probability of obtaining heads on a biased coin is 0.4. The coin is tossed 600 times.

(a) Write down the mean number of heads. [1 mark]

(b) Find the probability that the number of heads obtained is less than 250. [3 marks]

**3.** The time it takes for a student to complete a task is normally distributed with a mean of 20 minutes and a standard deviation of 1.25 minutes.

(a) A student is randomly selected. Find the probability that the student completes the task in less than 21.8 minutes. [2 marks]

(b) The probability that a student takes between $k$ and 21.8 minutes is 0.3. Find the value of $k$. [5 marks]

**4.** A multiple choice test has 20 questions and each question has four answers from which to choose. Each question has only one correct answer. Let $X$ be the number of correct answers chosen if the answer to each question is randomly guessed.

(a) Briefly explain why $X$ can be modelled by a binomial distribution. [2 marks]

(b) Find the mean of $X$. [2 marks]

(c) With random guessing on the test, find the probability that a student gets exactly three correct answers. [3 marks]

(d) With random guessing on the test, find the probability that a student gets at least three correct answers. [3 marks]

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**5.** Quentin trains his dog Rusty to run an obstacle course. Each day Rusty runs the course once and Quentin records how long it takes Rusty to run the course. The time it takes Rusty to run the course can be modelled by a normal distribution with a mean of 92 seconds and a standard deviation of 10 seconds. Quentin gives Rusty a treat if he runs the course in less than 80 seconds.

(a) Find the probability that Rusty gets a treat for running the course next Monday. [2 marks]

(b) Find the probability that Rusty gets a treat at least twice during the next week (Monday to Sunday). [4 marks]

**6.** The masses of a toy truck made in a certain factory are normally distributed with mean $\mu$ and standard deviation $\sigma$. 5% of the trucks have a mass greater than 85 grams and 10% have a mass less than 25 grams.

(a) Find the values of $\mu$ and $\sigma$. [5 marks]

(b) 75% of the masses are between the values of $\mu - k$ grams and $\mu + k$ grams. Find the value of $k$. [3 marks]

**7.** The mass $M$ of apples in grams is normally distributed with mean $\mu$. The table below shows probabilities for values of $M$.

| Values of $M$ | $M < 93$ | $93 \leq M \leq 119$ | $M > 119$ |
|---|---|---|---|
| $\mathrm{P}(M)$ | $k$ | 0.98 | 0.01 |

(a) (i) Write down the value of $k$.

(ii) Show that $\mu = 106$. [4 marks]

(b) Find $\mathrm{P}(M < 95)$. [4 marks]

The apples are packed in bags containing ten apples each. Any apples with a mass less than 95 grams are classified as small.

(c) Find the probability that a bag of apples selected at random contains at most one small apple. [3 marks]

(d) A crate contains 50 bags of apples. A crate is selected at random.

(i) Find the expected number of bags in this crate that contain at least one small apple.

(ii) Find the probability that at least 48 bags in this crate contain at most one small apple. [5 marks]