Test (SL) – Probability Distributions SOLUTION KEY

total marks on test: **55**

Calculator allowed on all questions

1. Solution:

(a) $0.05+k+0.5+0.3=1 \quad \Rightarrow \quad k=1-0.85 \quad \Rightarrow \quad k=0.15$ [2]

(b) $\mathrm{E}(X)=\sum x\cdot P(X=x)=0(0.05)+1(0.15)+2(0.5)+3(0.3) \quad \Rightarrow \quad \mathrm{E}(X)=2.05$ [2]

2. Solution:

(a) $\text{mean}=0.4(600)=240$ [1]

(b) $P(X\le 299)\approx 0.809$ [3]

3. Solution:

(a) $\mu=20,\ \sigma=1.25 \qquad P(X<21.8)\approx 0.925$ [2]

(b) $P(k<X<21.8)=0.3 \quad \Rightarrow \quad P(X<k)=0.925-0.3=0.625$

$P(X<k)=0.625 \quad \Rightarrow \quad z$ value for k: $z\approx 0.31864...$

$z=\dfrac{x-\mu}{\sigma} \quad \Rightarrow \quad 0.31864...=\dfrac{k-20}{1.25} \quad \Rightarrow \quad k=1.25(0.31864...)+20 \quad \Rightarrow \quad k\approx 20.4 \text{ min}$ [5]

4. Solution:

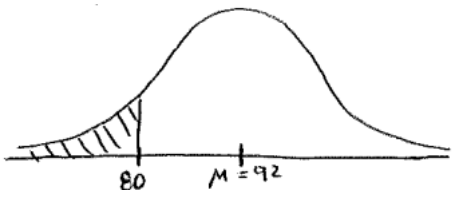
(a) There are repeated trials (20 questions) where there are only two outcomes (correct or incorrect) for each trial (question). Thus, $X\sim \mathrm{B}\left(20,\dfrac{1}{4}\right)$. [2]

(b) $\text{mean}=np=20\left(\dfrac{1}{4}\right)=5$ [2]

(c) $P(X=3)\approx 0.1338956... \quad \Rightarrow \quad P(X=3)\approx 0.134$ [3]

(d) $P(X\ge 3)\approx 0.9087396... \quad \Rightarrow \quad P(X\ge 3)\approx 0.909$ [3]

5. Solution:



(a) T is random variable representing time it takes Rusty to run the course.

$P(T<80)\approx 0.1150697... \quad \Rightarrow \quad P(T<80)\approx 0.115$ [2]

(b) X is random variable representing number of treats that Rusty earns during a 7-day week.

$X\sim \mathrm{B}(7,0.1150697...)$

$P(X\ge 2)=1-\left[P(X=0)+P(X=1)\right]\approx 0.188$ [4]

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6. Solution:

(a) $P(X > 85) = 0.05$ $\quad P(X < 25) = 0.10$

OR $P(X < 85) = 0.95$ $\quad P(X < Z) = 0.10$

$P(X < Z) = 0.95$

Z value for $0.95 \approx 1.6448$; and z value for $0.10 \approx -1.2816$

$Z = \frac{X-\mu}{\sigma}$; $1.6448 = \frac{85-\mu}{\sigma}$ $\quad -1.2816 = \frac{25-\mu}{\sigma}$

$1.6448\sigma = 85-\mu$ $\quad -1.2816\sigma = 25-\mu$

$\mu = 85 - 1.6448\sigma$ $\quad \mu = 25 + 1.2816\sigma$

$85 - 1.6448\sigma = 25 + 1.2816\sigma$

$60 = 2.9264\sigma$

$\sigma = \frac{60}{2.9264} \approx \underline{20.50}$

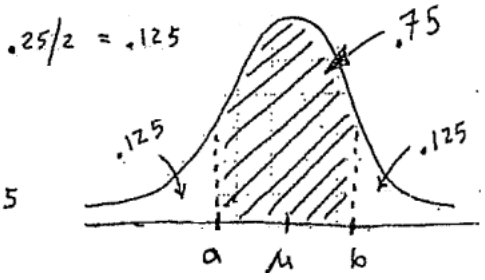
$\mu = 85 - 1.6448(20.50) \approx \underline{51.28}$

$\mu \approx 51.3$
$\sigma \approx 20.5$

[5]

(b) $P(\mu - K \leq X \leq \mu + K) = 0.75$

OR $P(X \leq \mu + K) = 0.75 + 0.125 = 0.875$



Z value for $0.875 \approx 1.15035$; and z value for $0.125 \approx -1.15035$

using $Z = \frac{X-\mu}{\sigma}$ $(\mu \approx 51.28,\ \sigma \approx 20.50)$ gives

$X = \mu + K \approx 74.86$
and $X = \mu - K \approx 27.70$
$\Big\}\ 2K \approx 47.16 \Rightarrow \underline{K \approx 23.6}$

[3]

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7. Solution:

(a) (i) $k+0.98+0.01=1 \Rightarrow k=0.01$.

(ii) because $\mathrm{P}(M<93)=0.01$ and $\mathrm{P}(M>119)=0.01$, then $M=93$ and $M=119$ are symmetric about the mean μ; hence, $\mu=\frac{93+119}{2} \Rightarrow \mu=106$ ***Q.E.D.*** [4]

(b) $\mathrm{P}(M>119)=0.01$; z value for 119: $z\approx 2.32634...$

$$z=\frac{x-\mu}{\sigma} \Rightarrow 2.32634...=\frac{119-106}{\sigma} \Rightarrow \sigma=\frac{13}{2.32634...}\approx 5.588177...$$

$M \sim \mathrm{N}(106, 5.588177...) \Rightarrow \mathrm{P}(M<95)\approx 0.0245$ [4]

(c) Let X be the random variable that represents # of apples in a bag of 10 apples that has a mass less than 95 grams (i.e. a small apple). $X \sim \mathrm{B}(10, 0.0245086...)$

$\mathrm{P}(X\le 1)=\mathrm{P}(X=0)+\mathrm{P}(X=1)\approx 0.78025...+0.19603...\approx 0.97628...$

Therefore, the probability that a bag of 10 apples selected at random has at least one small apple is approximately 0.976 [3]

(d) (i) Let Y be the random variable that represents # of bags in a crate of 50 bags that contains at most one small apple (i.e. less than 95 g). $Y \sim \mathrm{B}(50, 0.97628...)$

$\mathrm{E}(Y)=np\approx 50(0.97628...)\approx 48.8$ bags

(ii) $\mathrm{P}(Y\ge 48)\approx 0.88468...$

Therefore, the probability that a crate of 50 bags selected at random has at least 48 bags containing at most one small apple is approximately 0.885 [5]