Probability & Probability Distributions

10 Questions – calculator allowed on all questions (answers included)

1. A continuous random variable X has probability density function

$$f(x)=\begin{cases} kx, & 0\le x<3 \\ 3k, & 3\le x\le 5 \\ 0, & \text{otherwise} \end{cases}$$

Find:

(a) the value of k;

(b) the variance of X;

(c) the median of X.

2. Forty percent of the students at a school ride to and from school on bicycles. A group of ten students is chosen at random. Find the probability that:

(a) exactly three of the students ride a bicycle to school;

(b) at least three of the students ride a bicycle to school.

3. The mean weight of 400 male students at a school is 75 kg with a standard deviation of 7 kg. Assume that the weights are normally distributed.

(a) Find the number of students whose weight, to the nearest kg, lies between 64 and 80 kg.

(b) Find the number of students whose weight, to the nearest kg, is greater than 90 kg.

4. A continuous random variable X is normally distributed. 15% of the values of X are greater than 100 and 25% of the values of X are less than 70. Find the mean and standard deviation of X.

5. A continuous random variable X has a probability density function given by

$$p(x)=\begin{cases} 0, & x\le 0 \\ \frac{1}{6}x, & 0\le x<3 \\ 2-\frac{1}{2}x, & 3\le x\le 4 \\ 0, & x\ge 4 \end{cases}$$

(a) Show that the mean of X is $\frac{7}{3}$ and find the variance of X.

(b) Given that the median of X is $\sqrt{k}$, find the value of k.

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6. The probability of obtaining heads on a biased coin is 0.18. The coin is tossed seven times.

(a) Find the probability of obtaining **exactly** two heads.

(b) Find the probability of obtaining **at least** two heads.

7. In a certain country the men have heights which are normally distributed with mean 1.70 m and standard deviation 0.1 m. Five men are selected at random from this country. Find the probability that at least two of the five men have heights greater than 1.82 m.

8. The discrete random variable X has the following probability distribution

$$P(X = x) = \begin{cases} \frac{k}{x}, & x = 1, 2, 3, 4 \\ 0, & \text{otherwise} \end{cases}$$

(a) Find the value of the constant k.

(b) Find $E(X)$.

9. A continuous random variable X has probability density function $f(x)$ where

$$f(x) = \begin{cases} \frac{1}{2}e^{x/2}, & 0 \le x \le k \\ 0, & \text{otherwise} \end{cases}$$

(a) Find the **exact** value of k.

(b) Find the **exact** value of the median.

10. The masses of sacks of grain packed by an automatic machine loader are normally distributed.

(a) Given that 8% of the sacks have a mass greater than 50.8 kg, and 6% of the sacks have a mass less than 49.9 kg, find the mean mass and the standard deviation.

(b) The machine is now reset so that the mean is 50.5 kg and the standard deviation is 0.4 kg. Find the percentage of sacks that will have a mass less than 50 kg.

(c) With the mean and standard deviation from part (b), a farmer buys five of these sacks of grain. Find the probability that at least one of the sacks with have a mass greater than 51 kg.

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Answers

1. (a) $k = \frac{2}{21}$ (b) $\text{Var}(X) \approx 1.38$ (c) median $= \frac{13}{4}$

2. (a) $P(X=3) \approx 0.215$ (b) $P(X \geq 3) \approx 0.833$

3. (a) approximately 282 (b) approximately 6

4. mean ≈ 81.8, standard deviation ≈ 17.5

5. (a) $\text{Var}(X) = \frac{13}{18} = 0.7\bar{2} \approx 0.722$ (b) $k = 6$ [median $= \sqrt{6}$]

6. (a) $P(H=2) \approx 0.252$ (b) $P(H \geq 2) \approx 0.368$

7. $P(x \geq 2) \approx 0.104$

8. (a) $k = \frac{12}{25} = 0.48$ (b) $E(X) = \frac{48}{25} = 1.92$

9. (a) $k = \ln 4$ (b) median $= \ln\left(\frac{9}{4}\right)$

10. (a) mean ≈ 50.4 kg, standard deviation ≈ 0.304 kg
    (b) $\approx 10.6\%$
    (c) $P(X \geq 1) \approx 0.428$