Probability Distributions

► **formulas for discrete and continuous random variables** ◄

- expected value of a **discrete** random variable X $\quad \mathrm{E}(X) = \mu = \sum_x x \cdot \mathrm{P}(X = x)$
- expected value of a **continuous** random variable X: $\quad \mathrm{E}(X) = \mu = \int_{-\infty}^{\infty} x \cdot f(x)\,dx$
- variance: $\quad \mathrm{Var}(X) = \mathrm{E}(X-\mu)^2 = \mathrm{E}(X^2) - [\mathrm{E}(X)]^2$
- variance of a **discrete** random variable X: $\quad \mathrm{Var}(X) = \sum (x-\mu)^2 \cdot \mathrm{P}(X = x)$
  $= \sum x^2 \cdot \mathrm{P}(X = x) - \mu^2$
- variance of a **continuous** random variable X: $\quad \mathrm{Var}(X) = \int_{-\infty}^{\infty} (x-\mu)^2 \cdot f(x)\,dx$
  $= \int_{-\infty}^{\infty} x^2 \cdot f(x)\,dx - \mu^2$

Exercises

Qs 1-4: discrete probability distributions; **Qs 5-7**: continuous probability distributions

1. The discrete random variable X has a probability distribution function of $\mathrm{P}(X = x) = k(3-x)$ for $x = 0, 1, 2, 3$.

(a) Find the value of k.
(b) Display the probability distribution function of X as a table.
(c) Find $\mathrm{P}(1 \le X \le 3)$.
(d) Find $\mathrm{P}(1 < X < 3)$.
(e) Find $\mathrm{E}(X)$ and $\mathrm{Var}(X)$.

2. (a) Luke is playing a game that consists of shooting a basketball at a basketball hoop from a set distance. The probability that Luke will shoot the ball through the hoop at this distance is 0.6. A single turn at this game is a player shooting four times. A player wins a prize, if at the end of his/her turn, they have made at least three shots through the hoop. Luke takes a single turn at the game. What is the probability that Luke wins a prize?
(b) Another version of the game has players shooting the basketball from the same set distance, but now they win a prize if they can make three shots in at most five attempts. Luke takes a single turn at the game. What is the probability that Luke wins a prize?
(c) The games are over and Luke is practicing taking shots from the same set distance. He takes 12 practice shots. What is the probability that Luke makes eight of these shots?

3. There are 25 students in a class, of which 16 are boys and 9 are girls. A group of six students are selected at random. Calculate the probability that the group contains:
(a) three boys and three girls;
(b) students all of the same gender.

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4. (a) A game consists of tossing three fair coins. If exactly one head shows then you lose \$6. If exactly two heads show then there is no loss or gain. If all three coins show heads then you win \$2, and if all three coins show tails you win \$4. In playing this game, what amount of money is expected to be lost or won?

(b) The game is now changed so that the coin being tossed is not fair. The probability of the coin showing heads is now 0.7. In playing this game, what amount of money is expected to be lost or won?

(c) The game is changed back so that a fair coin is used, i.e. $\mathrm{P}(\text{Head}) = 0.5$. It is also decided that the game will now be a 'fair' game – that is, the expected win/loss is zero. If the only thing that is changed is how much money is won when all three coins show tails, then what must be the amount won for three tails showing for the game to be 'fair', i.e. expected win/loss equal to zero?

5. A continuous random variable X has the probability density function $f(x)$ defined as follows

$$f(x) = \begin{cases} \frac{3}{4}\left(x^2+1\right) & \text{for } 0 \le x \le 1 \\ 0 & \text{elsewhere} \end{cases}$$

(a) Find the expected value, $\mathrm{E}(X)$ or μ, and the standard deviation, σ, for X.

(b) Find $\mathrm{P}\left(|X-\mu| < \sigma\right)$

6. A continuous random variable X has the probability density function $p(x)$ defined as follows

$$p(x) = \begin{cases} ax & 0 \le x < 1 \\ a(2-x) & 1 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the value of the constant a.

(b) Find $\mathrm{E}(X)$.

(c) Find $\mathrm{Var}(X)$.

(d) $\mathrm{P}\left(\frac{3}{4} \le X \le \frac{3}{2}\right)$.

7. A continuous random variable X has probability density function defined as follows

$$f(x) = \begin{cases} 0 & x < 0 \\ kx & 0 \le x \le 2 \\ \frac{16k}{x^3} & x > 2 \end{cases}$$

(a) Find the value of the constant k.

(b) Find the **median** value of X.

(c) Find the value of a such that $\mathrm{P}(X > a) = 0.005$.

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Answers

1. (a) $k = \frac{1}{6}$ (b)

x	0	1	2	3
$\mathrm{P}(X = x)$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$	0

(c) $\mathrm{P}(1 \le X \le 3) = \frac{1}{2}$ (d) $\mathrm{P}(1 < X < 3) = \mathrm{P}(X = 2) = \frac{1}{6}$ (e) $\mathrm{E}(X) = \frac{2}{3}$, $\mathrm{Var}(X) = \frac{5}{9}$

2. (a) 0.4752 (b) 0.683 (c) 0.213

3. (a) 0.266 (b) 0.0457

4. (a) expect to **lose** \$1.50 (b) expect to **lose** \$0.34 (c) win \$16 when showing 3 tails

5. (a) $\mu = 0.5625, \quad \sigma \approx 0.289$ (b) $\mathrm{P}(0.852 < X < 0.273) \approx 0.583$

6. (a) $a = 1$ (b) $\mathrm{E}(X) = 1$ (c) $\mathrm{Var}(X) = \frac{1}{6}$ (d) $\mathrm{P}(\frac{3}{4} \le X \le \frac{3}{2}) = \frac{19}{32}$

7. (a) $k = \frac{1}{4}$ (b) median $= 2$ (c) $a = 20$