

DP Analysis & Approaches (SL/HL)

Maths with Lemon

Topic 7.3b The Normal Distribution

Mathematician: _____

SL 4.9	Guidance, clarification and syllabus links
The normal distribution and curve. Properties of the normal distribution. Diagrammatic representation.	Awareness of the natural occurrence of the normal distribution. Approximately 68% of the data lies between $\mu \pm \sigma$, 95% between $\mu \pm 2\sigma$, and 99.7% between $\mu \pm 3\sigma$.
Normal probability calculations.	Probabilities and values of the variable must be found using technology.
Inverse normal calculations.	For inverse normal calculations, mean and standard deviation will be given.
SL 4.12	Guidance, clarification and syllabus links
Standardization of normal variables (z -values).	The standardized value z gives the number of standard deviations from the mean.
Inverse normal calculations where mean and standard deviation are unknown.	Use z -values to calculate unknown means and standard deviations.

Warm Up

The following is a probability distribution for a random variable Y .

y	0	1	2	3
$P(Y = y)$	0.1	0.4	k	$(k - 1)^2$

(a) Find the value of k .

(b) Find the expected value.

The Normal Distribution

The normal distribution models variables with **symmetrical, bell-shaped** probability density functions.

The Anatomy of the Normal Distribution

The exact location and shape of a normal curve are determined by:

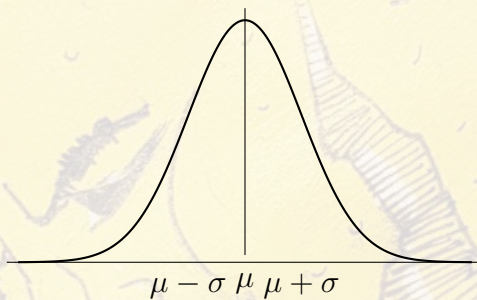
- the mean μ , which measures the centre;
- the standard deviation σ , which measures the spread.

If X is normally distributed with mean μ and standard deviation σ , write

$$X \sim N(\mu, \sigma^2).$$

Properties:

- symmetrical about $x = \mu$;
- $f(x) > 0$ for all x ;
- the x -axis is a horizontal asymptote;
- the maximum occurs at $x = \mu$.



Match the parameters

Match each pair with the appropriate normal curve.

- 1) $\mu = 15, \sigma = 2$
- 2) $\mu = 15, \sigma = 0.5$
- 3) $\mu = 5, \sigma = 1$
- 4) $\mu = 5, \sigma = 3$

Using Approximate Distributions (Empirical Rule)

Approximately

68% lies within $\mu \pm \sigma$

95% lies within $\mu \pm 2\sigma$

99.7% lies within $\mu \pm 3\sigma$.

Suppose the heights of men aged 20–29 in the United States are normally distributed with mean 176 cm and standard deviation 7.42 cm.

(a) Find the percentage between 161.16 cm and 183.42 cm.

(b) Find the percentage shorter than 190.84 cm.

A catch of fish has mean weight 1.8 kg and standard deviation 0.5 kg:

$$X \sim N(1.8, 0.5^2).$$

Find an interval centred around the mean containing approximately 68% of the fish.

Suppose X is normally distributed with mean 30 and standard deviation 5.

(a) State the value which is:

- (i) 2 standard deviations above the mean;
- (ii) 1 standard deviation below the mean.

(b) Describe 35, 20 and 45 in terms of standard deviations above or below the mean.

(c) Draw a curve to illustrate the distribution.

(d) What proportion of values are between 25 and 30?

(e) Find the probability that a randomly selected value is between 35 and 40.

Using Symmetry and Total Probability

$P(X < n)$ means the probability that a value X is less than n . It can also be interpreted as the proportion of the distribution below n .

If

$$X \sim N(12, \sigma^2),$$

$$P(X < 11) = 0.369 \quad \text{and} \quad P(X > 14) = 0.252,$$

find:

- (a) $P(X > 11)$,
- (b) $P(11 < X < 14)$,
- (c) the approximate percentile of $X = 14$.

Paper 1 Normal Distribution Problems

Problem 1

X is normally distributed with mean 90. The area to the left of 84 is 0.22.

- (a) Write down $P(X < 84)$.
- (b) Find $P(84 < X < 90)$.
- (c) Find $P(84 < X < 96)$.

Problem 2

X is normally distributed with mean 120. The area between 105 and 135 is 0.4.

- (a) Write down $P(105 < X < 135)$.
- (b) Find $P(X < 135)$.
- (c) Find $P(X > 105 \mid X < 135)$.

Problem 3

X is normally distributed with mean 50 and standard deviation 8.

- (a) Shade the region representing $P(X > 58)$.
- (b) Given $P(X > 66) = P(X < k)$, find k .
- (c) Given $P(X > 66) = 0.0228$, find $P(k < X < 66)$.

Problem 4

Two normal distributions are

$$X \sim N(10, 3^2), \quad Y \sim N(14, 3^2).$$

It is given that

$$P(X > 11.6) = 0.3.$$

- (a) Find k such that $P(Y > k) = 0.3$.
- (b) Find $P(8.4 < X < 11.6)$.

Problem 5

Let

$$X \sim N(75, 5^2).$$

- (a) Given $P(X < 65) = P(X > a)$, find a .
- (b) Given $P(65 < X < a) = 0.954$, find $P(X > a)$.



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Mathematics made clear.