DP Analysis & Approaches (SL/HL)

Maths with Lemon

Topic 7.1b Discrete Random Variables

Mathematician: ____________________________________

SL 4.7	Guidance, clarification and syllabus links
Concept of discrete random variables and their probability distributions. Expected value (mean), for discrete data. Applications.	Probability distributions may be given in a table or by a formula. $$\sum P(X=x)=1$$ A fair game has $$E(X)=0$$ when X represents the gain of a player.

Warm Up

In an IB school, 50 students take an art subject. Each student takes either dance, film, or music.

	Dance	Film	Music
Male	6	9	7
Female	11	7	10

Two students are selected at random.

(a) Find the probability that neither student studies dance.

(b) Find the probability that both students study music.

Three students are selected at random.

(c) Find the probability that the three students study different art subjects.

Overview

The set of all possible outcomes of an experiment is called the **sample space**. The probabilities of all outcomes in a probability distribution must add to 1.
A **discrete random variable** takes distinct, countable values. Examples include:

- number of heads obtained in coin tosses,
- number of seeds germinating,
- number of calls received per minute,
- number of boxes containing fewer than 100 matches.

Coin Tossing

A fair coin is tossed three times. Let X be the number of heads.

$$X \in \{0, 1, 2, 3\}.$$

The probability distribution is

x	0	1	2	3
$P(X = x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

For example,

$$P(X = 2) = \frac{3}{8}.$$

Write:
(a) A statement for the probability that 0 heads are obtained.

(b) A statement for the probability that 3 heads are obtained.

Formal Conditions

For every possible value x_i,

$$\boxed{0 \leq P(X = x_i) \leq 1}$$

and

$$\boxed{\sum_{i=1}^{N} P(X = x_i) = 1}.$$

Example 1

The number of targets missed by a biathlete has the probability distribution

x	0	1	2	3	4
$P(X = x)$	0.25	0.15	0.15	k	0.25

Find k.

Example 2

A random variable has

$$P(X = i) = \frac{k}{2i}, \qquad i = 1, 2, 3, 4, 5, 6.$$

Find k.

Example 3

A random variable X has distribution

x	1	2	3	4
$P(X = x)$	α	4α	9α	16α

Find α.

Example 4

A random variable X has distribution

x	0	1	2	3
$P(X = x)$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{5}$	k

Find k and $P(X \leq 2)$.

Calculating Expected Value

The expected value, or expected mean, is the long-run average value of a random variable.

$$\boxed{E(X) = \mu = \sum_{i=1}^{N} x_i P(X = x_i)}$$

Example 5

For the biathlon distribution

x	0	1	2	3	4
$P(X = x)$	0.25	0.15	0.15	0.20	0.25

find the expected number of targets missed.

Fair Games

If X represents a player's **net gain**, a fair game has

$$\boxed{E(X) = 0}.$$

Example 6

A game uses a 20-sided die. If Player A rolls a prime number, Player A wins 5 points. Otherwise Player B wins k points. If X represents how many points Player A is winning by, find k so that the game is fair.

Example 7

A game has the following payout distribution.

Payout x_i	0	-1	2	3
$P(x_i)$	$\frac{4}{15}$	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{1}{30}$

Is the game fair?

Example 8

Given

$$E(X) = 2.5$$

and

x	1	2	3	4
$P(X = x)$	0.2	p	q	0.2

find p and q.

Homework

Problem 1. Two dice are rolled. If their sum is prime, the score is 0; otherwise the score is the sum. Find the expected score.

Problem 2. Javier designs a fair game with outcomes Lose, Tie, Win and Bonus. Their probabilities are respectively

$$3y, \quad 2y, \quad 4y, \quad 10y^2,$$

and their payouts are

$$k, \quad 0, \quad 1, \quad 2.$$

Find the value of k that makes the game fair.

Problem 3. A biased die has $P(6) = 0.4$. The other five numbers are equally probable. Find $P(1)$.

Problem 4. A bag contains 12 black, 18 yellow and 30 purple marbles. Joe pays \$10 to pick one marble. If black is selected, he receives \$20; if yellow, \$15; if purple, \$k. Find k so that the game is fair.

Problem 5. The distribution of X is

x	0	1	2
$P(X = x)$	ak	ak^2	ak^3

Given

$$E(X) = \frac{5}{13},$$

find

$$P(X > 0 \mid X < 2).$$

Problem 6. The distribution of X is

x	1	2	3	4	5
$P(X = x)$	p	q	r	$2r$	$3r$

Given

$$P(X \geq 3) = 0.6 \qquad \text{and} \qquad E(X) = 3.15,$$

find

$$P(2 \leq X \leq 4).$$

Maths with Lemon

Mathematics made clear.