DP Analysis & Approaches (SL/HL)

Maths with Lemon

Lesson 4.3 – Measures of Dispersion

Mathematician: _____

Warm Up: Grandparent Visits

The number of visits and frequencies are:

Visits	0	1	2	3	4	5	6	7	8
Year 6	0	0	3	6	10	8	5	0	0
Year 10	5	8	7	2	4	0	4	3	1

For each class calculate (a) the five-number summary, (b) the mean, (c) the range and IQR. Then compare the typical number of visits and variation.

1. Outliers: Medicine Bottles

Two manufacturers provide 200 bottles each. Use the frequency tables to determine quartiles, outlier fences and any outliers. Draw modified box plots on the same scale.

	Advint		Tylenar
Pills	Frequency	Pills	Frequency
95	1	92	1
96	3	96	1
97	2	97	13
98	25	98	23
99	32	99	34
100	40	100	42
101	34	101	33
102	22	102	22
103	19	103	19
104	12	104	10
105	8	106	1
106	2	108	1

2. Understanding Standard Deviation

For $2, 3, 5, 9, 11$, calculate deviations from the mean, their sum, the mean squared deviation, and the standard deviation.

3. Comparing Orange Suppliers

Blemished oranges per crate: Sunblessed $4, 16, 14, 8, 8$; Valencia Star $9, 12, 11, 10, 13$. Find the mean and standard deviation of each and compare.

4. Guess the Goat's Mass

Nine guesses (kg): $18.3, 21.6, 22.2, 24.0, 20.4, 17.4, 14.9, 26.7, 23.1$. Find the mean, standard deviation and interpret the latter.

5. Choosing a Route

Times in minutes: Motorway $15, 16, 20, 28, 21$; Country roads $19, 21, 20, 22, 18$. Find the mean and standard deviation of each; recommend a route with reasons.

6. Standard Deviation from a Frequency Table

IB score	1	2	3	4	5	6	7
Frequency	2	5	6	20	16	11	3

For the 63 students calculate $\sum fx$, $\sum fx^2$, the mean and standard deviation. Verify with technology.

7. Linear Transformations

(a) Find mean and standard deviation of $3, 6, 7, 9, 10$. (b)–(d) Predict, calculate and comment after adding 4 to every value. (e) For $3, 6, 7, 9, x$, the standard deviation is $\sqrt{34}$. Find both possible values of x and the corresponding mean y. (f)–(i) Describe how multiplication and addition affect the mean and standard deviation.

8. Checking Values Near the Mean

For $1, 1, 3, 4, 5, 6, 7, 7, 8, 11, 12, 13, 13, 14, 15, 15$, how many values are within one standard deviation of the mean?

9. Challenge 1

An ordered set $\{a, b, c, d, e\}$ has mode 1, median 5, range 7 and mean 4.4. Find its standard deviation.

10. Challenge 2

A set has mean 5 and standard deviation 1.3. Every value is multiplied by a, then c^2 is added. The new mean is 3 and standard deviation 2.6. Find a and all possible real values of c.

Detailed Markscheme

Topic 4.3 – Measures of Dispersion

Convention: population standard deviation σ (divide by n); quartiles are medians of the lower and upper halves.

Warm Up

Year 6 has $N = 32$ observations and total $\sum fx = 134$, hence $\bar{x} = 4.1875$. Its five-number summary is $(2, 3, 4, 5, 6)$, range 4, IQR 2. Year 10 has $N = 34$, $\sum fx = 97$, hence $\bar{x} = 97/34 \approx 2.85$. Its five-number summary is $(0, 1, 2, 4, 8)$, range 8, IQR 3. Year 6 visited more often on average, while Year 10 shows greater variation.

1. Outliers

Advint: $Q_1 = 99$, median 100, $Q_3 = 102$, IQR 3. Fences 94.5 and 106.5: no outliers. Whiskers 95 to 106.
Tylenar: $Q_1 = 99$, median 100, $Q_3 = 102$, IQR 3. Fences 94.5 and 106.5: outliers 92 and 108. Whiskers 96 to 106; plot 92 and 108 as separate points.

2. Deviations

Mean $\bar{x} = 6$. Deviations $-4, -3, -1, 3, 5$, sum 0; squared deviations $16, 9, 1, 9, 25$, sum 60. Thus

$$\boxed{\sigma^2 = 12, \qquad \sigma = \sqrt{12} \approx 3.46}.$$

3. Orange Suppliers

Sunblessed: $\bar{x} = 10$, squared deviations $36, 36, 16, 4, 4$, sum 96. Hence $\sigma = \sqrt{96/5} \approx 4.38$.
Valencia Star: $\bar{x} = 11$, squared deviations $4, 1, 0, 1, 4$, sum 10. Hence $\sigma = \sqrt{10/5} = \sqrt{2} \approx 1.41$.
Valencia Star has a slightly higher mean number of blemishes but much more consistent crates; Sunblessed has fewer blemishes on average.

4. Goat

The total is 188.6 kg and $\bar{x} = 188.6/9 \approx 20.96$ kg. Using $\sigma = \sqrt{\sum (x - \bar{x})^2/9}$ gives $\boxed{\sigma \approx 3.42 \text{ kg}}$.
Guesses typically differ from the mean by about 3.39 kg.

5. Route Choice

Motorway: $\bar{x} = 20$ min, squared deviations $25 + 16 + 0 + 64 + 1 = 106$, $\sigma = \sqrt{106/5} \approx 4.60$ min.
Country roads: $\bar{x} = 20$ min, squared deviations $1+1+0+4+4 = 10$, $\sigma = \sqrt{2} \approx 1.41$ min. Recommend country roads: same mean, much smaller variation.

6. Frequency Table

$$\sum f = 63, \qquad \sum fx = \boxed{277}, \qquad \sum fx^2 = \boxed{1339}.$$

$$\bar{x} = \frac{277}{63} \approx 4.397, \quad \sigma^2 = \frac{1339}{63} - \left(\frac{277}{63}\right)^2, \quad \boxed{\sigma \approx 1.39}.$$

7. Transformations

(a) Original mean 7; squared deviations $16 + 1 + 0 + 4 + 9 = 30$, so $\sigma = \sqrt{6} \approx 2.45$.
(b)–(d) Adding 4 gives $7, 10, 11, 13, 14$, mean 11, unchanged standard deviation $\sqrt{6}$.
(e) Let $y = (25 + x)/5$. From $\sigma^2 = 34$:

$$\frac{(3 - y)^2 + (6 - y)^2 + (7 - y)^2 + (9 - y)^2 + (x - y)^2}{5} = 34.$$

Solving gives $\boxed{x = 20 \text{ or } -\frac{15}{2}}$ and $\boxed{y = 9 \text{ or } \frac{7}{2}}$, with matching signs.
(f) Multiplication by a multiplies mean by a. (g) Addition of b adds b to mean. (h) Multiplication by a multiplies SD by $|a|$. (i) Adding a constant leaves SD unchanged.

8. Values Within One

SD

The mean is $\bar{x} = \frac{135}{16} = 8.4375$; $\sigma \approx 4.74$. The interval $[\bar{x} - \sigma, \bar{x} + \sigma]$ is approximately $[3.69, 13.18]$. There are $\boxed{10}$ values within this interval: $4, 5, 6, 7, 7, 8, 11, 12, 13, 13$.

9. Challenge 1

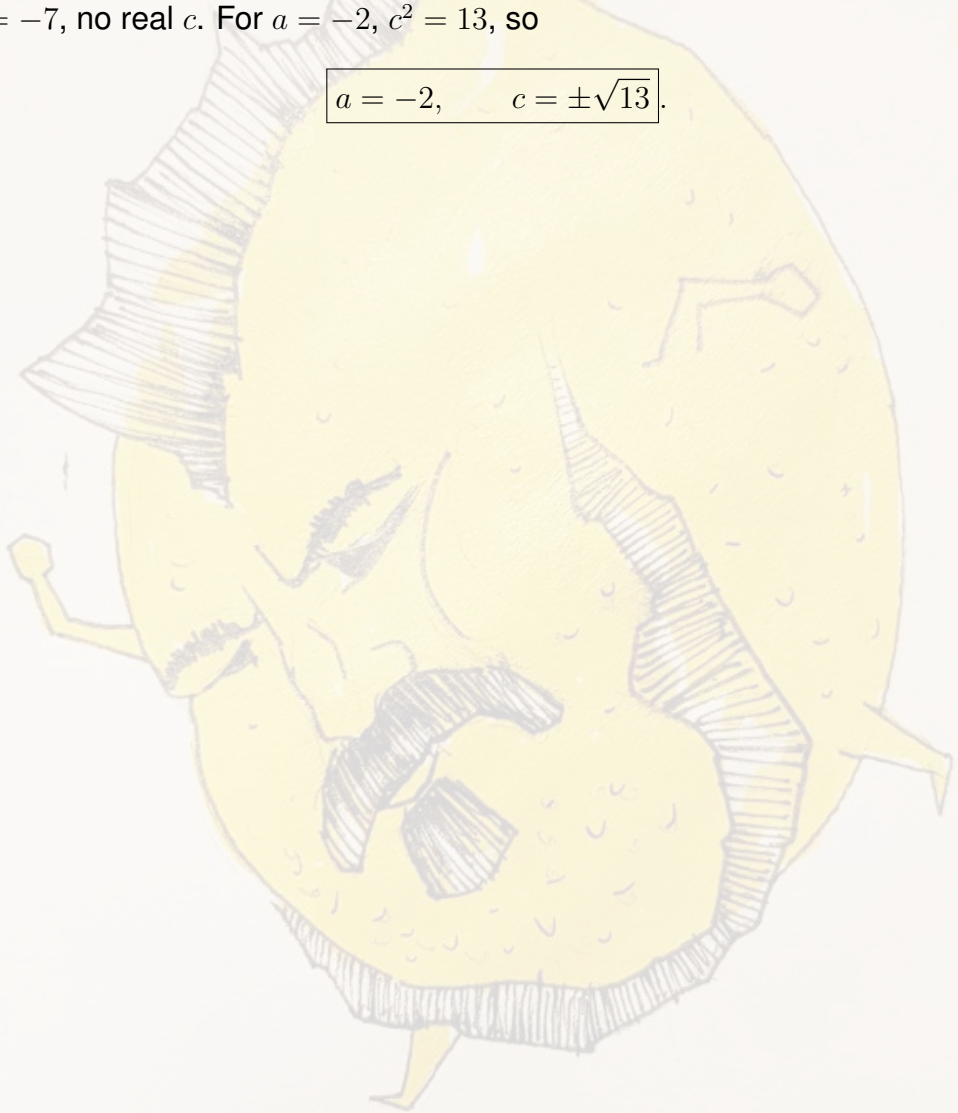
Mode 1 and median 5 imply the ordered set begins $1, 1, 5$. The range 7 implies $e = 8$; the sum is $5(4.4) = 22$, so $d = 7$. Thus the set is $1, 1, 5, 7, 8$.

$$\bar{x} = 4.4, \quad \sigma = \sqrt{\frac{(1-4.4)^2 + (1-4.4)^2 + (5-4.4)^2 + (7-4.4)^2 + (8-4.4)^2}{5}} \approx \boxed{2.94}.$$

10. Challenge 2

The new standard deviation is $|a|(1.3) = 2.6$, so $a = 2$ or $a = -2$. The new mean satisfies $5a + c^2 = 3$. For $a = 2$, $c^2 = -7$, no real c. For $a = -2$, $c^2 = 13$, so

$$\boxed{a = -2, \qquad c = \pm\sqrt{13}}.$$





Maths with Lemon

Mathematics made clear.