DP Analysis & Approaches (SL/HL)

Maths with Lemon

Lesson 5.2 – Using the Unit Circle

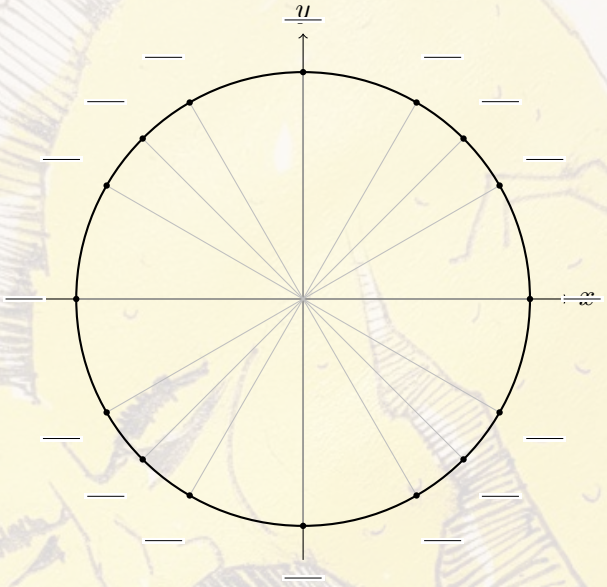
Mathematician: _____

Syllabus links: SL 3.4 and AHL 3.4

SL 3.4	Guidance and clarification
Definitions of $\sin\theta$, $\cos\theta$ from the unit circle; $\tan\theta = \sin\theta/\cos\theta$; exact ratios of special angles.	Relationships across quadrants, e.g. $\cos(-x) = \cos x$, $\sin(\pi + x) = -\sin x$, $\tan(3\pi - x) = -\tan x$.
AHL 3.4: relationships between angles in different quadrants	

Warm Up – Fill in the Unit Circle

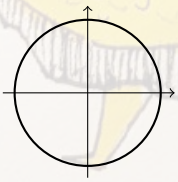
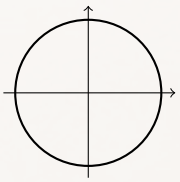
Fill the **degree measures**, **radian measures**, and coordinates $(\cos\theta, \sin\theta)$ for all 16 marked angles. Use exact values.



Remember: $(x, y) = (\cos\theta, \sin\theta)$ and $\tan\theta = y/x$ when $x \neq 0$.

Finding Trigonometric Ratios

The left column is solved. Use the same method to complete the right column.

Worked Example	Your Turn
1. $\theta = \dfrac{\pi}{4}$ 	**2.** $\theta = -\dfrac{7\pi}{6}$ 
Quadrant I; reference angle $\pi/4$.	Find a coterminal angle and its quadrant.
$\sin\theta = \boxed{\dfrac{\sqrt{2}}{2}}$, $\cos\theta = \boxed{\dfrac{\sqrt{2}}{2}}$	$\sin\theta =$ _____
$\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \boxed{1}$.	$\cos\theta =$ _____
	$\tan\theta =$ _____
	Show your working.

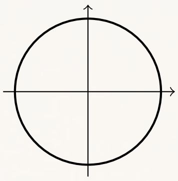
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Worked Example

3. $\theta = 390°$



$390° - 360° = 30°$ (quadrant I).

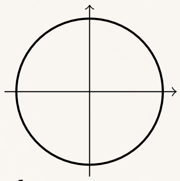
$$\sin 390° = \sin 30° = \boxed{\frac{1}{2}}$$

$$\cos 390° = \cos 30° = \boxed{\frac{\sqrt{3}}{2}}$$

$$\frac{1/2}{\sqrt{3}/2} = \boxed{\frac{\sqrt{3}}{3}}$$

Your Turn

4. $\theta = 135°$



Identify the quadrant and reference angle.

$\sin\theta =$ __________

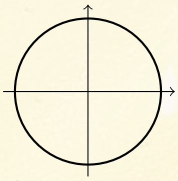
$\cos\theta =$ __________

$\tan\theta =$ __________

Show your working.

Worked Example

5. $\theta = 690°$



$690° - 360° = 330°$ (quadrant IV). Reference angle $30°$.

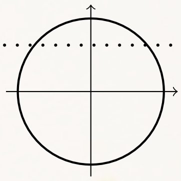
$$\sin 690° = \boxed{-\frac{1}{2}}$$

$$\cos 690° = \boxed{\frac{\sqrt{3}}{2}}$$

$$\tan 690° = \boxed{-\frac{\sqrt{3}}{3}}.$$

Your Turn

6. $\theta = -120°$



Find a coterminal angle and its quadrant.

$\sin\theta =$ __________

$\cos\theta =$ __________

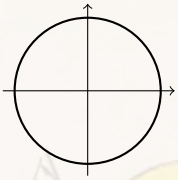
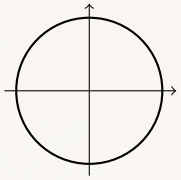
$\tan\theta =$ __________

Show your working.

Reciprocal Trigonometric Functions – HL Only

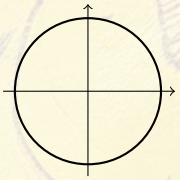
$$\csc\theta = \frac{1}{\sin\theta}, \qquad \sec\theta = \frac{1}{\cos\theta}, \qquad \cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}.$$

Reciprocal ratios are undefined when their denominators are zero.

Worked Example (HL)	**Your Turn (HL)**
7. $\theta = \frac{\pi}{3}$ 	**8.** $\theta = 225°$ 
Quadrant I; reference angle $\pi/3$.	$\sin\theta =$ _____, $\csc\theta =$ _____,
$\sin\theta = \frac{\sqrt{3}}{2}$, $\csc\theta = \frac{2}{\sqrt{3}} = \boxed{\frac{2\sqrt{3}}{3}}$,	$\cos\theta =$ _____, $\sec\theta =$ _____,
$\cos\theta = \frac{1}{2}$, $\sec\theta = \boxed{2}$,	$\tan\theta =$ _____, $\cot\theta =$ _____.
$\tan\theta = \sqrt{3}$, $\cot\theta = \boxed{\frac{\sqrt{3}}{3}}$.	*Show your working.*
	. . . . .
	. . . . .
	. . . . .

9. Your Turn – HL

For $\theta = -\frac{5\pi}{4}$, find all six exact trigonometric ratios. Sketch the angle on the unit circle.



$\sin\theta =$ _____, $\csc\theta =$ _____,

$\cos\theta =$ _____, $\sec\theta =$ _____,

$\tan\theta =$ _____, $\cot\theta =$ _____.

10. Complete the Exact-Values Table

Radians	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	π
Degrees		30°				
$\sin\theta$			$\sqrt{2}/2$			
$\cos\theta$				1/2		
$\tan\theta$					Undefined	
$\csc\theta$						
$\sec\theta$						
$\cot\theta$						

Detailed Markscheme

Lesson 5.2 – Using the Unit Circle

Warm Up

The angles in degrees, radians and exact coordinates, in counterclockwise order:

$0°$	0	$(1,0)$	$30°$	$\pi/6$	$(\sqrt{3}/2, 1/2)$
$45°$	$\pi/4$	$(\sqrt{2}/2, \sqrt{2}/2)$	$60°$	$\pi/3$	$(1/2, \sqrt{3}/2)$
$90°$	$\pi/2$	$(0,1)$	$120°$	$2\pi/3$	$(-1/2, \sqrt{3}/2)$
$135°$	$3\pi/4$	$(-\sqrt{2}/2, \sqrt{2}/2)$	$150°$	$5\pi/6$	$(-\sqrt{3}/2, 1/2)$
$180°$	π	$(-1,0)$	$210°$	$7\pi/6$	$(-\sqrt{3}/2, -1/2)$
$225°$	$5\pi/4$	$(-\sqrt{2}/2, -\sqrt{2}/2)$	$240°$	$4\pi/3$	$(-1/2, -\sqrt{3}/2)$
$270°$	$3\pi/2$	$(0,-1)$	$300°$	$5\pi/3$	$(1/2, -\sqrt{3}/2)$
$315°$	$7\pi/4$	$(\sqrt{2}/2, -\sqrt{2}/2)$	$330°$	$11\pi/6$	$(\sqrt{3}/2, -1/2)$

Trigonometric Ratios: Questions 1–6

1. $\theta = \pi/4$: $\sin\theta = \sqrt{2}/2$, $\cos\theta = \sqrt{2}/2$, $\tan\theta = 1$.

2. $-7\pi/6 + 2\pi = 5\pi/6$; quadrant II, reference angle $\pi/6$.

$$\boxed{\sin\theta = \frac{1}{2}, \quad \cos\theta = -\frac{\sqrt{3}}{2}, \quad \tan\theta = -\frac{\sqrt{3}}{3}}.$$

3. $390° \equiv 30° \pmod{360°}$: $\sin\theta = 1/2$, $\cos\theta = \sqrt{3}/2$, $\tan\theta = \sqrt{3}/3$.

4. $135° = 180° - 45°$; quadrant II:

$$\boxed{\sin\theta = \frac{\sqrt{2}}{2}, \quad \cos\theta = -\frac{\sqrt{2}}{2}, \quad \tan\theta = -1}.$$

5. $690° \equiv 330°$; quadrant IV: $\sin\theta = -1/2$, $\cos\theta = \sqrt{3}/2$, $\tan\theta = -\sqrt{3}/3$.

6. $-120° + 360° = 240°$; quadrant III, reference $60°$:

$$\boxed{\sin\theta = -\frac{\sqrt{3}}{2}, \quad \cos\theta = -\frac{1}{2}, \quad \tan\theta = \sqrt{3}}.$$

Reciprocal Ratios: Questions 7–9

7. $\pi/3$: $\sin = \sqrt{3}/2$, $\cos = 1/2$, $\tan = \sqrt{3}$; $\csc = 2\sqrt{3}/3$, $\sec = 2$, $\cot = \sqrt{3}/3$.

8. $225° = 180° + 45°$, quadrant III:

$$\begin{aligned} \sin 225° &= -\frac{\sqrt{2}}{2}, & \csc 225° &= -\sqrt{2}, \\ \cos 225° &= -\frac{\sqrt{2}}{2}, & \sec 225° &= -\sqrt{2}, \\ \tan 225° &= 1, & \cot 225° &= 1. \end{aligned}$$

9. $-5\pi/4 + 2\pi = 3\pi/4$, quadrant II:

$$\begin{aligned} \sin\theta &= \frac{\sqrt{2}}{2}, & \csc\theta &= \sqrt{2}, \\ \cos\theta &= -\frac{\sqrt{2}}{2}, & \sec\theta &= -\sqrt{2}, \\ \tan\theta &= -1, & \cot\theta &= -1. \end{aligned}$$

10. Exact-Values Table – Completed

Radians	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	π
Degrees	$0°$	$30°$	$45°$	$60°$	$90°$	$180°$
$\sin\theta$	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1	0
$\cos\theta$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0	-1
$\tan\theta$	0	$\sqrt{3}/3$	1	$\sqrt{3}$	Undefined	0
$\csc\theta$	Undefined	2	$\sqrt{2}$	$2\sqrt{3}/3$	1	Undefined
$\sec\theta$	1	$2\sqrt{3}/3$	$\sqrt{2}$	2	Undefined	-1
$\cot\theta$	Undefined	$\sqrt{3}$	1	$\sqrt{3}/3$	0	Undefined



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Mathematics made clear.