Kinematics [219 marks]

1. [Maximum mark: 7]

The displacement, in centimetres, of a particle from an origin, O, at time t seconds, is given by $s(t) = t^2 \cos t + 2t \sin t$, $0 \leq t \leq 5$.

(a) Find the maximum distance of the particle from O. [3]

(b) Find the acceleration of the particle at the instant it first changes direction. [4]

2. [Maximum mark: 5]

A particle moves in a straight line such that its velocity, $v\ \text{ms}^{-1}$, at time t seconds is given by

$$v = 4t^2 - 6t + 9 - 2\sin(4t),\ 0 \leq t \leq 1.$$

The particle's acceleration is zero at $t = T$.

(a) Find the value of T. [2]

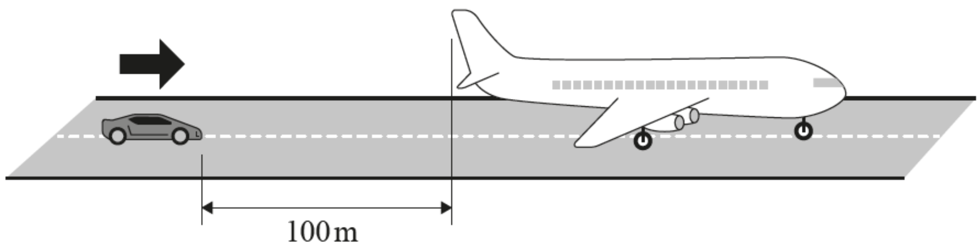
(b) Let s_1 be the distance travelled by the particle from $t = 0$ to $t = T$ and let s_2 be the distance travelled by the particle from $t = T$ to $t = 1$.

Show that $s_2 > s_1$. [3]

3. [Maximum mark: 17]

An airplane lands on a runway 100 metres in front of a stationary car. At the instant the airplane lands, the car begins to travel in the same direction towards the airplane.

diagram not to scale



Let t represent the number of seconds after the airplane lands. For $t \geq 0$, the velocities of the airplane and the car in $\mathrm{m\,s^{-1}}$, can be modelled by the following equations:

$$v_{\text{air}} = 60\mathrm{e}^{-0.1t}$$

$$v_{\text{car}} = 5t$$

(a) When the airplane lands, write down the speed of

(a.i) the airplane; [1]

(a.ii) the car. [1]

(b) Find

(b.i) the value of t when the airplane and the car have the same speed; [2]

(b.ii) the speed at this time. [1]

Let $d(t)$ represent the distance, in metres, between the car and the back of the airplane after t seconds.

(c) If $d(0) = 100$, find $d(t)$. [7]

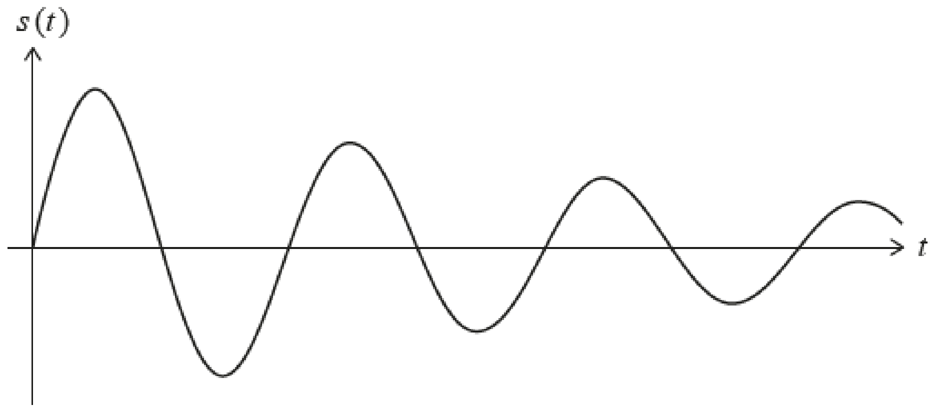
(d) Hence, find how long it takes for the car to reach the back of the airplane. [2]

(e) Find the distance travelled by the car when it reaches the back of the airplane. [3]

4. [Maximum mark: 17]

A particle P moves in a straight line so that its displacement, $s\ \mathrm{cm}$, from a fixed point O at time t seconds is given by $s(t) = 2^{\left(1-\frac{t}{5}\right)}\ \sin\left(\frac{2\pi t}{3}\right)$, where $t \geq 0$.

The following diagram shows part of the graph of $y = s(t)$.



(a) Find

(a.i) the maximum displacement of P from O; [2]

(a.ii) the maximum velocity of P. [3]

(b) Find

(b.i) the minimum value of the displacement function $s(t)$; [2]

(b.ii) the displacement of P from O when $t = 3.5$. [1]

(c) Hence, determine the **total distance** travelled by P in the first 3.5 seconds. [3]

The first time that P returns and passes through O is when $t = T$.

(d) Write down the value of T. [1]

The particle passes through O every T seconds.

A sequence $u_1,\ u_2,\ u_3 \ldots$ is formed where $u_1,\ u_2,\ u_3 \ldots$ are the largest **distances** from O in each of the intervals $0 < t < T,\ T < t < 2T,\ 2T < t < 3T \ldots$ respectively.

It is known that $u_1,\ u_2,\ u_3 \ldots$ form a geometric sequence.

(e.i) Determine the value of the common ratio r of this geometric sequence. [2]

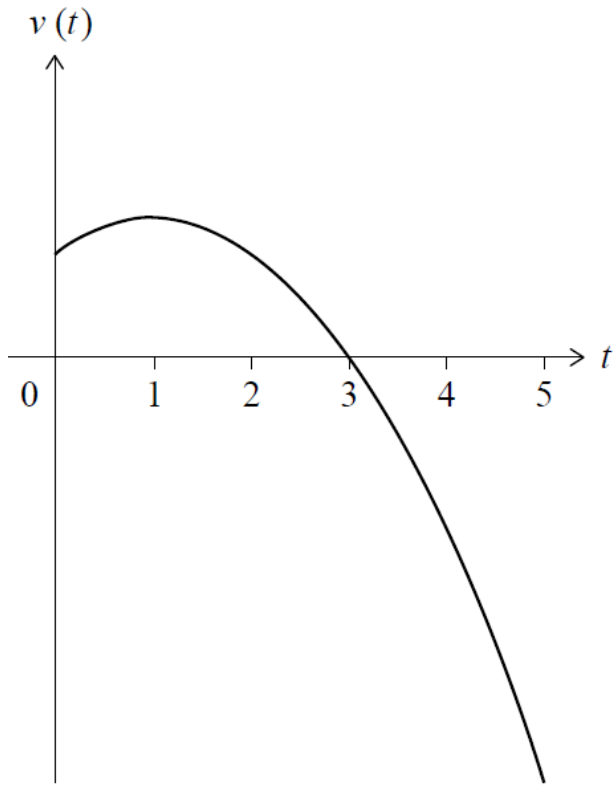
(e.ii) Calculate the **total distance** travelled by the particle if it were to continue to move in this way indefinitely. [3]

5. [Maximum mark: 13]

An object moves in a straight line.

Its velocity $v\,\mathrm{m\,s^{-1}}$, at time t seconds, is given by $v(t) = 30 + 20t - 10t^2$ for $0 \leq t \leq 5$.

The graph of v is shown in the following diagram.



The graph of v has a local maximum point where $t = 1$ and intersects the t-axis at $t = 3$.

(a) Determine the object's

(a.i) maximum velocity; [2]

(a.ii) maximum speed. [2]

At $t = T$, the object changes direction.

(b.i) Write down the value of T. [1]

(b.ii) Find the distance travelled by the object in the first T seconds. [4]

(c) Determine whether the object returns to its initial position during the time period $0 \leq t \leq 5$, justifying your answer. [4]

6. [Maximum mark: 7]

A particle P moves in a straight line. The velocity $v\,\mathrm{m\,s^{-1}}$ of P, at time t seconds is given by $v(t) = \mathrm{e}^{-\sin t}\cos(2t)$, for $0 \le t \le 5$.

(a) Find the maximum speed of P. [2]

(b) Find the total distance travelled by P. [2]

(c) Find the acceleration when P changes direction for the **second** time. [3]

7. [Maximum mark: 14]

Two athletes, Fiona and Lucy, compete in a 200 metres race along a straight track.

Fiona's velocity, in $\mathrm{m\,s^{-1}}$, during the race can be modelled by $v(t) = \frac{8.14t}{\sqrt{t^2+0.2}}$, where $t \ge 0$. Time, t, is measured in seconds from when the race starts.

(a.i) Write down the value of $v(1)$. [1]

(a.ii) Find the time when Fiona's velocity is $5\,\mathrm{m\,s^{-1}}$. [2]

(b) Find the time when Fiona's acceleration is $4\,\mathrm{m\,s^{-2}}$. [2]

(c.i) Write down the limit of $v(t)$ as t approaches infinity. [2]

(c.ii) State a reason why the value in part (c)(i) is not valid in the context of this question [1]

Lucy's velocity, in $\mathrm{m\,s^{-1}}$, during the race can be modelled by $w(t) = \frac{8t}{\sqrt{t^2+0.3}}$, where $t \ge 0$.

Fiona completes the race and crosses the finishing line in front of Lucy.

(d) Find the distance Lucy is from the finishing line when Fiona completes the 200 metres. [6]

8. [Maximum mark: 16]

In this question all values of x and t are in radians.

Consider the function $f(x) = 3\ \sin\ (4\pi x)$.

(a.i) Write down the amplitude of the graph of f. [1]

(a.ii) Find the period of f. [2]

Consider a second function $g(x) = -4\ \cos\ (4\pi x)$.

The sum of these functions can be expressed in the form $f(x) + g(x) = a\ \cos\ (b(x - c))$, where $a,\ b,\ c > 0$.

(b) By considering the graph of $y = f(x) + g(x)$, determine

(b.i) the value of a; [2]

(b.ii) the value of b; [1]

(b.iii) the smallest possible value of c. [1]

A car is travelling along a straight residential street with speed bumps placed at regular intervals on the road to encourage safer driving. The car travels at a minimum velocity when passing over speed bumps and reaches a maximum velocity in between speed bumps.

Its velocity, in $\mathrm{m\,s^{-1}}$, can be modelled by the function $v(t) = -3.5\ \cos\ \left(\frac{\pi}{14}(t - 5)\right) + 9$, where t is measured in seconds.

(c) Find the time at which the car first reaches its maximum velocity.

[1]

(d) Find the number of speed bumps the car passes over in the first two minutes of motion. [1]

(e.i) Find $v\prime(t)$. [2]

(e.ii) Hence, or otherwise, write down the maximum acceleration of the car.

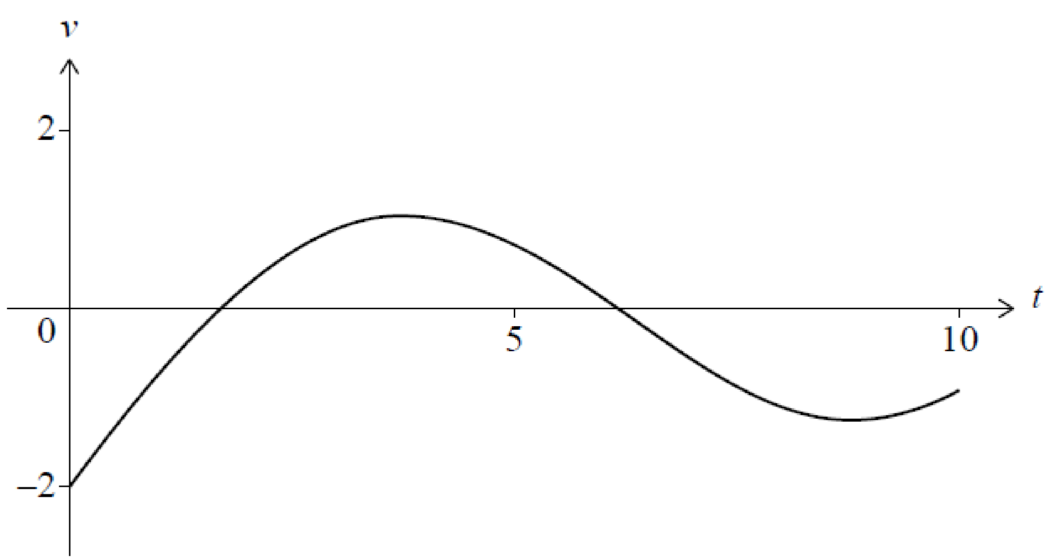
[2]

(f) Find the distance, in metres, between consecutive speed bumps. [3]

9. [Maximum mark: 6]

A particle moves in a straight line such that it passes through a fixed point O at time $t = 0$, where t represents time measured in seconds after passing O. For $0 \leq t \leq 10$ its velocity, v metres per second, is given by $v = 2\sin(0.5t) + 0.3t - 2$.

The graph of v is shown in the following diagram.



(a) Find the smallest value of t when the particle changes direction. [2]

The displacement of the particle is measured in metres from O.

(b) Find the range of values of t for which the velocity is positive. [2]

(c) Find the displacement of the particle relative to O when $t = 10$. [2]

10. [Maximum mark: 6]

A particle moves in a straight line such that its velocity, $v\ \mathrm{ms}^{-1}$, at time t seconds is given by $v(t) = 1 + \mathrm{e}^{-t} - \mathrm{e}^{-\sin 2t}$ for $0 \le t \le 2$.

(a) Find the velocity of the particle at $t = 2$. [1]

(b) Find the maximum velocity of the particle. [2]

(c) Find the acceleration of the particle at the instant it changes direction. [3]

11. [Maximum mark: 5]

A particle moves along a straight line. Its displacement, s metres, from a fixed point O after time t seconds is given by $s(t) = 5.2 \sin\left(\sqrt{4t+6}\right)$, where $0 \le t \le 10$.

The particle first comes to rest after q seconds.

(a) Find the value of q. [2]

(b) Find the total distance that the particle travels in the first q seconds.

[3]

12. [Maximum mark: 16]

A farmer is growing a field of wheat plants. The height, H cm, of each plant can be modelled by a normal distribution with mean μ and standard deviation σ.

It is known that $\mathrm{P}(H < 94.6) = 0.288$ and $\mathrm{P}(H > 98.1) = 0.434$.

(a) Find the probability that the height of a randomly selected plant is between 94.6 cm and 98.1 cm. [2]

(b) Find the value of μ and the value of σ. [5]

The farmer measures 100 randomly selected plants. Any plant with a height greater than 98.1 cm is considered ready to harvest. Heights of plants are independent of each other.

(c.i) Find the probability that exactly 34 plants are ready to harvest. [2]

(c.ii) Given that fewer than 49 plants are ready to harvest, find the probability that exactly 34 plants are ready to harvest. [4]

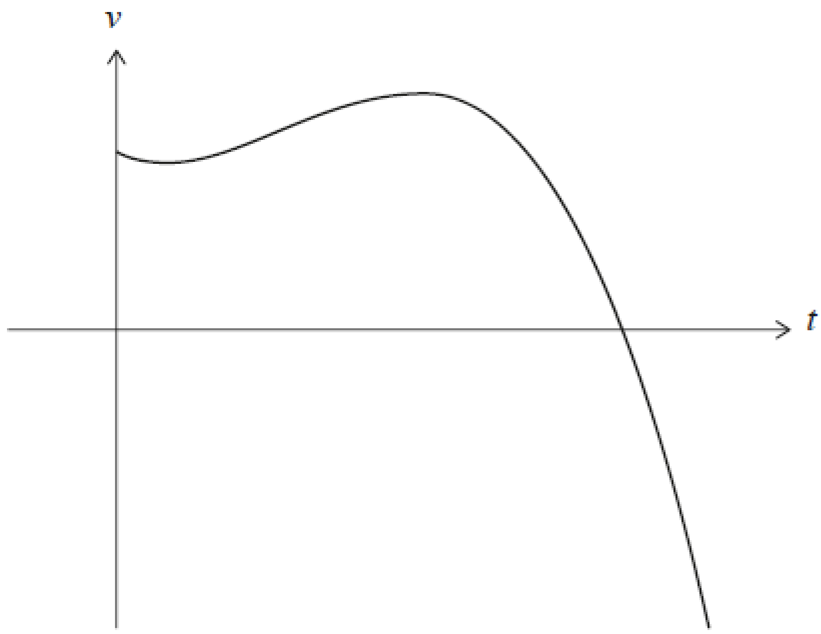
In another field, the farmer is growing the same variety of wheat, but is using a different fertilizer. The heights of these plants, F cm, are normally distributed with mean 98.6 and standard deviation d. The farmer finds the interquartile range to be 4.82 cm.

(d) Find the value of d. [3]

13. [Maximum mark: 17]

An object moves along a straight line. Its velocity, $v\,\mathrm{m\,s^{-1}}$, at time t seconds is given by $v(t) = -t^3 + \frac{7}{2}t^2 - 2t + 6$, for $0 \le t \le 4$. The object first comes to rest at $t = k$.

The graph of v is shown in the following diagram.



At $t = 0$, the object is at the origin.

(a) Find the displacement of the object from the origin at $t = 1$. [5]

(b) Find an expression for the acceleration of the object. [2]

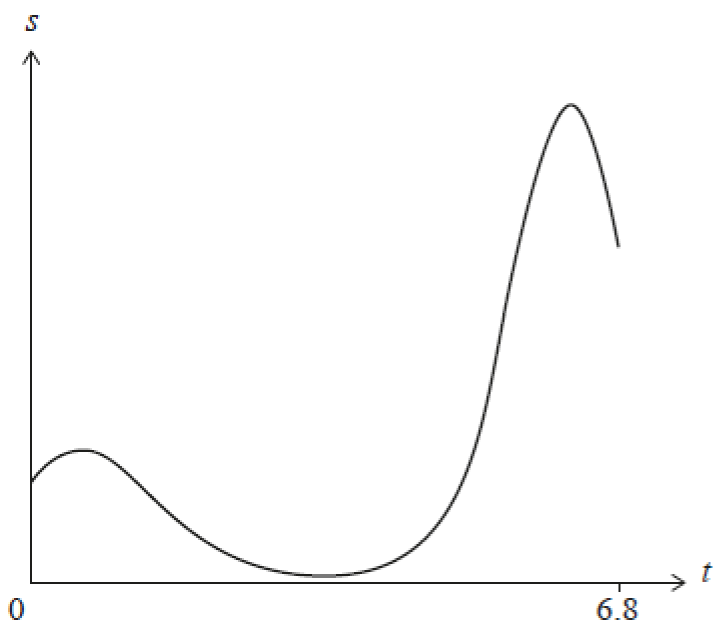
(c) Hence, find the greatest speed reached by the object before it comes to rest. [5]

(d) Find the greatest speed reached by the object for $0 \leq t \leq 4$. [2]

(e) Write down an expression that represents the distance travelled by the object while its speed is increasing. Do not evaluate the expression. [3]

14. [Maximum mark: 16]

A particle moves in a straight line. Its displacement, s metres, from a fixed point P at time t seconds is given by $s\left(t\right) = 3(t+2)^{\cos t}$, for $0 \le t \le 6.8$, as shown in the following graph.



(a) Find the particle's initial displacement from the point P. [2]

(b) Find the particle's velocity when $t = 2$. [2]

(c) Determine the intervals of time when the particle is moving away from the point P. [5]

The acceleration of the particle is zero when $t = b$ and $t = c$, where $b < c$.

(d) Find the value of b and the value of c. [4]

(e) Find the total distance travelled by the particle for $b \le t \le c$. [3]

15. [Maximum mark: 7]

A particle moves along a straight line so that its velocity, $v\,\mathrm{m\,s}^{-1}$, after t seconds is given by $v(t) = \mathrm{e}^{\sin t} + 4\sin t$ for $0 \le t \le 6$.

(a) Find the value of t when the particle is at rest. [2]

(b) Find the acceleration of the particle when it changes direction. [3]

(c) Find the total distance travelled by the particle. [2]

16. [Maximum mark: 7]

A particle moves in a straight line such that its velocity, $v\,\mathrm{m\,s^{-1}}$, at time t seconds is given by $v = \frac{(t^2+1)\cos t}{4},\ 0 \le t \le 3$.

(a) Determine when the particle changes its direction of motion. [2]

(b) Find the times when the particle's acceleration is $-1.9\,\mathrm{m\,s^{-2}}$. [3]

(c) Find the particle's acceleration when its speed is at its greatest. [2]

17. [Maximum mark: 16]

A particle P moves along the x-axis. The velocity of P is $v\,\mathrm{m\,s^{-1}}$ at time t seconds, where $v(t) = 4 + 4t - 3t^2$ for $0 \le t \le 3$. When $t = 0,\ P$ is at the origin O.

(a.i) Find the value of t when P reaches its maximum velocity. [2]

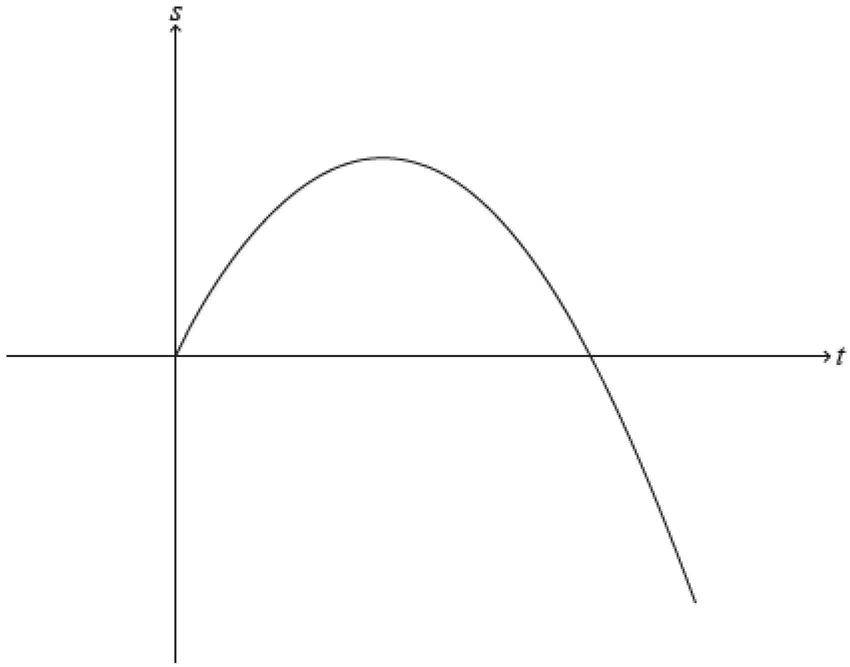
(a.ii) Show that the distance of P from O at this time is $\frac{88}{27}$ metres. [5]

(b) Sketch a graph of v against t, clearly showing any points of intersection with the axes. [4]

(c) Find the total distance travelled by P. [5]

18. [Maximum mark: 14]

Particle A travels in a straight line such that its displacement, s metres, from a fixed origin after t seconds is given by $s(t) = 8t - t^2$, for $0 \leq t \leq 10$, as shown in the following diagram.



Particle A starts at the origin and passes through the origin again when $t = p$.

(a) Find the value of p. [2]

Particle A changes direction when $t = q$.

(b.i) Find the value of q. [2]

(b.ii) Find the displacement of particle A from the origin when $t = q$
. [2]

(c) Find the distance of particle A from the origin when $t = 10$. [2]

The total distance travelled by particle A is given by d.

(d) Find the value of d. [2]

(e) A second particle, particle B, travels along the same straight line such that its velocity is given by $v(t) = 14 - 2t$, for $t \geq 0$.

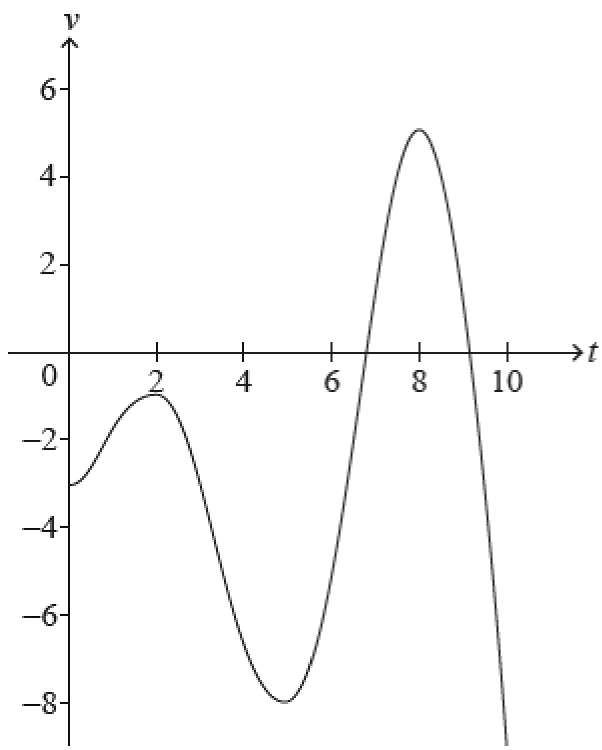
When $t = k$, the distance travelled by particle B is equal to d.

Find the value of k. [4]

19. [Maximum mark: 6]

A particle moves in a straight line. The velocity, $v \text{ ms}^{-1}$, of the particle at time t seconds is given by $v(t) = t \sin t - 3$, for $0 \leq t \leq 10$.

The following diagram shows the graph of v.



(a) Find the smallest value of t for which the particle is at rest. [2]

(b) Find the total distance travelled by the particle. [2]

(c) Find the acceleration of the particle when $t = 7$. [2]

20. [Maximum mark: 7]

In this question, all lengths are in metres and time is in seconds.

Consider two particles, P_1 and P_2, which start to move at the same time.

Particle P_1 moves in a straight line such that its displacement from a fixed-point is given by $s(t) = 10 - \frac{7}{4}t^2$, for $t \geq 0$.

(a) Find an expression for the velocity of P_1 at time t. [2]

(b) Particle P_2 also moves in a straight line. The position of P_2 is given by $\boldsymbol{r} = \begin{pmatrix} -1 \\ 6 \end{pmatrix} + t\begin{pmatrix} 4 \\ -3 \end{pmatrix}$.

The speed of P_1 is greater than the speed of P_2 when $t > q$.

Find the value of q. [5]