Kinematics [219 marks]

1. [Maximum mark: 7]

The displacement, in centimetres, of a particle from an origin, O, at time t seconds, is given by $s(t) = t^2 \cos t + 2t \sin t$, $0 \le t \le 5$.

(a) Find the maximum distance of the particle from O. [3]

Markscheme

use of a graph to find the coordinates of the local minimum ***(M1)***

$s = -16.513...$ ***(A1)***

maximum distance is 16.5 cm (to the left of O) ***A1***

[3 marks]

(b) Find the acceleration of the particle at the instant it first changes direction. [4]

Markscheme

attempt to find time when particle changes direction *eg* considering the first maximum on the graph of s or the first t – intercept on the graph of s'. ***(M1)***

$t = 1.51986...$ ***(A1)***

attempt to find the gradient of s' for **their** value of t, $s''(1.51986...)$ ***(M1)***

$=-8.92$ (cm/s^2) ***A1***

[4 marks]

2. [Maximum mark: 5]

A particle moves in a straight line such that its velocity, $v\ \text{ms}^{-1}$, at time t seconds is given by

$$v = 4t^2 - 6t + 9 - 2\sin(4t),\ 0 \le t \le 1.$$

The particle's acceleration is zero at $t = T$.

(a) Find the value of T. [2]

Markscheme

* This sample question was produced by experienced DP mathematics senior examiners to aid teachers in preparing for external assessment in the new MAA course. There may be minor differences in formatting

compared to formal exam papers.

attempts either graphical or symbolic means to find the value of t when $\frac{dv}{dt} = 0$ **(M1)**

$T = 0.465\,(\mathrm{s})$ **A1**

[2 marks]

(b) Let s_1 be the distance travelled by the particle from $t = 0$ to $t = T$ and let s_2 be the distance travelled by the particle from $t = T$ to $t = 1$.

Show that $s_2 > s_1$. [3]

Markscheme

attempts to find the value of either $s_1 = \int_0^{0.46494\ldots} v\,dt$ or $s_2 = \int_{0.46494\ldots}^{1} v\,dt$ **(M1)**

$s_1 = 3.02758\ldots$ and $s_2 = 3.47892\ldots$ **A1A1**

Note: Award as above for obtaining, for example, $s_2 - s_1 = 0.45133\ldots$ or $\frac{s_2}{s_1} = 1.14907\ldots$

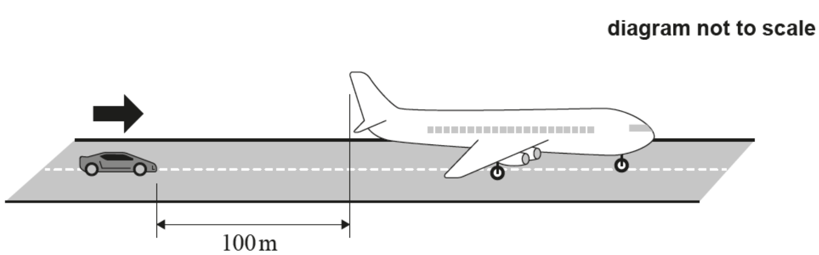
Note: Award a maximum of **M1A1A0FT** for use of an incorrect value of T from part (a).

so $s_2 > s_1$ **AG**

[3 marks]

3. [Maximum mark: 17]

An airplane lands on a runway 100 metres in front of a stationary car. At the instant the airplane lands, the car begins to travel in the same direction towards the airplane.



Let t represent the number of seconds after the airplane lands. For $t \geq 0$, the velocities of the airplane and the car in $\mathrm{m\,s^{-1}}$, can be modelled by the following equations:

$v_{\text{air}} = 60\mathrm{e}^{-0.1t}$

$v_{\text{car}} = 5t$

(a) When the airplane lands, write down the speed of

(a.i) the airplane; [1]

Markscheme

speed of airplane $= 60\,(\mathrm{m\,s^{-1}})$ ***A1***

[1 mark]

(a.ii) the car. [1]

Markscheme

speed of car $= 0\,(\mathrm{m\,s^{-1}})$ ***A1***

[1 mark]

(b) Find

(b.i) the value of t when the airplane and the car have the same speed; [2]

Markscheme

recognizing $v_{\text{air}} = v_{\text{car}}$ when speeds are the same ***(M1)***

$6.35564\ldots$

$t = 6.36\,(\mathrm{s})$ ***A1***

[2 marks]

(b.ii) the speed at this time. [1]

Markscheme

$31.7782\ldots$

speed $= 31.8\,(\mathrm{m\,s^{-1}})$ ***A1***

[1 mark]

Let $d(t)$ represent the distance, in metres, between the car and the back of the airplane after t seconds.

(c) If $d(0) = 100$, find $d(t)$. [7]

Markscheme

let $s_{\text{air}}(t)$ be the displacement of the airplane and $s_{\text{car}}(t)$ be the displacement of the car at time t

recognizing to integrate either velocity function ***(M1)***

$\int 60\mathrm{e}^{-0.1t}\ \mathrm{d}\ t$, $\int 5t\ \mathrm{d}\ t$

correct integration ***(A1)(A1)***

$-600\mathrm{e}^{-0.1t},\ \frac{5}{2}t^2(+C)$ (award ***A1*** for each term)

recognize to subtract displacements (seen anywhere) ***(M1)***

EITHER

substituting initial condition into their $d(t)$ ***(M1)***

$100 = -600\mathrm{e}^{-1(0)} - \frac{5}{2} \times 0^2 + C$ **OR** $-100 = 600\mathrm{e}^{-1(0)} + \frac{5}{2} \times 0^2 + C$ (or equivalent)

OR

substituting initial condition into their $s_{\text{air}}(t)$ ***(M1)***

$(s_{\text{car}}(0) = 0 \Rightarrow)100 = -600\mathrm{e}^{-1(0)} + C$

THEN

$C = 700$ (or -700) ***(A1)***

$d(t) = -600\mathrm{e}^{-0.1t} - \frac{5}{2}t^2 + 700$ (accept $d(t) = \left|600\mathrm{e}^{-0.1t} + \frac{5}{2}t^2 - 700\right|$) ***A1***

[7 marks]

(d) Hence, find how long it takes for the car to reach the back of the airplane. [2]

Markscheme

recognizing to solve their $d(t) = 0$ ***(M1)***

$15.0586\ldots$

$t = 15.1\,(\mathrm{s})$ ***A1***

[2 marks]

(e) Find the distance travelled by the car when it reaches the back of the airplane. [3]

Markscheme

attempt to find the distance travelled by the car ***(M1)***

$\int_0^{15.0586} 5t\,\mathrm{d}t$, $\frac{5}{2}(15.0586)^2$ ***(A1)***

$566.904\ldots$

$567\,(\mathrm{m})$ (accept $570\,(\mathrm{m})$ from 3 sf answer in (d)) ***A1***

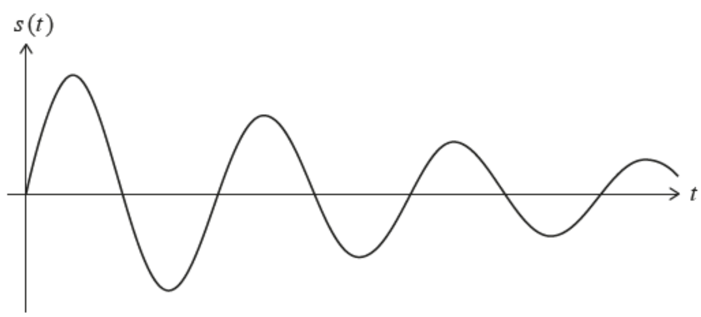
Note: Award ***(M1)(A1FT)A0***, where appropriate, if the candidate has an incorrect $d(t)$ and/or incorrect value of t which leads to their distance travelled by the car to be less than $100\,(\mathrm{m})$.
For example, where the candidate finds $d(t) = 600\mathrm{e}^{0.1t} - \frac{5}{2}t^2 - 500$ in part (c), award ***(M1)(A1FT)A0*** to 7.08, or 7.06 from using 3 sf value.

[3 marks]

4. [Maximum mark: 17]

A particle P moves in a straight line so that its displacement, s cm, from a fixed point O at time t seconds is given by $s(t) = 2^{\left(1-\frac{t}{5}\right)} \sin\left(\frac{2\pi t}{3}\right)$, where $t \geq 0$.

The following diagram shows part of the graph of $y = s(t)$.



(a) Find

(a.i) the maximum displacement of P from O; [2]

Markscheme

attempt to find a maximum point ***(M1)***

$(x =)0.718442\ldots$ **OR** $(y =)1.80645\ldots$ **OR** $(x =)3.71844\ldots$ **OR** $(y =)1.19181\ldots$ etc

$1.80645\ldots$

(maximum displacement)$= 1.81\,(\text{cm})$ ***A1***

[2 marks]

(a.ii) the maximum velocity of P. [3]

Markscheme

recognition that $v = \frac{\mathrm{d}s}{\mathrm{d}t}$ ***(M1)***

recognition that the gradient at $t = 0$ represents the maximum velocity ***(M1)***

$v = \frac{\mathrm{d}s}{\mathrm{d}t}$ at $t = 0$

$v = 4.18879025\ldots \left(\frac{4\pi}{3}\right)$

$v = 4.19 \left(\frac{4\pi}{3}\right) (\text{cm s}^{-1})$ ***A1***

[3 marks]

(b) Find

(b.i) the minimum value of the displacement function $s(t)$; [2]

Markscheme

attempt to find a minimum point ***(M1)***

$(x =)2.21844\ldots$ **OR** $(y =) - 1.46729\ldots$ **OR** $(x =)5.21844\ldots$ **OR** $(y =) - 0.968053\ldots$

$-1.46729\ldots$

-1.47 ***A1***

[2 marks]

(b.ii) the displacement of P from O when $t = 3.5$. [1]

Markscheme

$(t = 3.5)\ s = 1.06620\ldots$

$s = 1.07$ ***A1***

[1 mark]

(c) Hence, determine the **total distance** travelled by P in the first 3.5 seconds. [3]

Markscheme

METHOD 1

attempt to sum **distances** from O involving maximum, minimum and distance at $t = 3.5$ ***(M1)***

$1.80645\ldots \times 2 + 1.46729\ldots \times 2 + 1.06620\ldots$ ***(A1)***

$= 7.61369\ldots$

$7.61\,(\mathrm{cm})$ ***A1***

METHOD 2

attempt to integrate the absolute value of $\frac{\mathrm{d}s}{\mathrm{d}t}$ ***(M1)***

$\int_0^{3.5} \left| \frac{\mathrm{d}}{\mathrm{d}t}\left(2^{1-\frac{t}{5}} \sin\left(\frac{2\pi t}{3}\right)\right)\right| \mathrm{d}t$ ***(A1)***

$= 7.61369\ldots$

$7.61\,(\mathrm{cm})$ ***A1***

[3 marks]

The first time that P returns and passes through O is when $t = T$.

(d) Write down the value of T. [1]

Markscheme

$(T =)1.5$ ***A1***

[1 mark]

The particle passes through O every T seconds.

A sequence $u_1,\ u_2,\ u_3\ldots$ is formed where $u_1,\ u_2,\ u_3\ldots$ are the largest **distances** from O in each of the intervals $0 < t < T,\ T < t < 2T,\ 2T < t < 3T\ldots$ respectively.

It is known that $u_1,\ u_2,\ u_3\ldots$ form a geometric sequence.

(e.i) Determine the value of the common ratio r of this geometric sequence. [2]

Markscheme

attempt to divide two of $u_1,\ u_2,\ u_3\ldots$ ***(M1)***

$\frac{u_2}{u_1}\left(= \frac{1.46729\ldots}{1.80645\ldots}\right)$ **OR** $\frac{u_3}{u_2}\left(= \frac{1.19181\ldots}{1.46729\ldots}\right)$ **OR** $r^2 = \frac{u_3}{u_1}\left(= \frac{1.19181\ldots}{1.80645\ldots}\right)$

$0.812252...$

$(r =)0.812$ ***A1***

[2 marks]

(e.ii) Calculate the **total distance** travelled by the particle if it were to continue to move in this way indefinitely. [3]

Markscheme

attempt to use the sum to infinity of the GP ***(M1)***

$S_{\infty} = \frac{1.80645...}{1-0.812252...} (= 9.62167...)$

recognition that the total distance is double their S_{∞} ***(M1)***

$2 \times S_{\infty} = 2 \times \frac{1.80645...}{1-0.812252...}$ **OR** $2 \times 9.62167...$

$19.2433...$

$19.2\,(\text{cm})$ ***A1***

Note: Award ***(M1)(M1)A0*** for values of r that lead to an answer that is smaller than $2 \times$ first maximum displacement (3.6129).

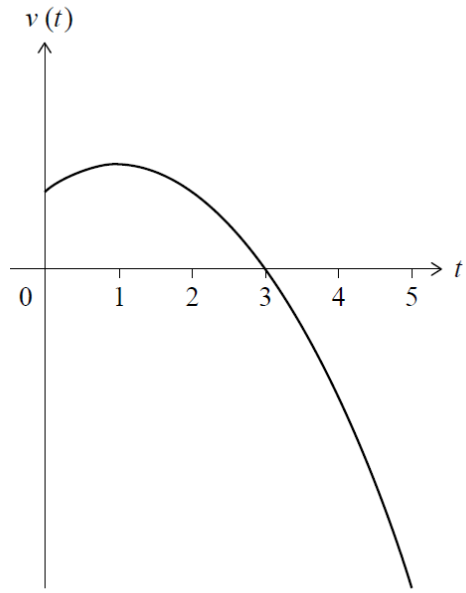
[3 marks]

5. [Maximum mark: 13]

An object moves in a straight line.

Its velocity $v\,\text{m}\,\text{s}^{-1}$, at time t seconds, is given by $v(t) = 30 + 20t - 10t^2$ for $0 \leq t \leq 5$.

The graph of v is shown in the following diagram.



The graph of v has a local maximum point where $t = 1$ and intersects the t-axis at $t = 3$.

(a) Determine the object's

(a.i) maximum velocity; [2]

Markscheme

attempt to find $v(1)$ ***(M1)***

$v(1) = 30 + 20 - 10$

max velocity $40\ (\text{ms}^{-1})$ ***A1***

[2 marks]

(a.ii) maximum speed. [2]

Markscheme

attempt to find $v(5)$ ***(M1)***

$v(5) = 30 + 20(5) - 10(5)^2 = -120$

max speed $120\ (\text{ms}^{-1})$ ***A1***

[2 marks]

At $t = T$, the object changes direction.

(b.i) Write down the value of T. [1]

Markscheme

$T = 3$ ***A1***

[1 mark]

(b.ii) Find the distance travelled by the object in the first T seconds. [4]

Markscheme

attempt to set up an integral of $v(t)$ to find area under velocity graph ***(M1)***

$$\int_0^3 \left(30 + 20t - 10t^2\right)\, \mathrm{d}\, t$$

$$\left[30t + 10t^2 - \frac{10t^3}{3}\right]_0^3$$ ***A1***

attempt to substitute their limits into their integrated function and subtract ***(M1)***

$$30(3) + 10(3)^2 - \frac{10(3)^3}{3} (= 90 + 90 - 90)$$

distance 90 (metres) ***A1***

[4 marks]

(c) Determine whether the object returns to its initial position during the time period $0 \le t \le 5$, justifying your answer. [4]

Markscheme

METHOD 1

attempt to find total displacement by setting up an integral of $v(t)$ between $t = 0$ and $t = 5$ ***(M1)***

$$\int_0^5 \left(30 + 20t - 10t^2\right)\, \mathrm{d}\, t$$

$$= \left[30t + 10t^2 - \frac{10t^3}{3}\right]_0^5$$ ***(A1)***

$$= 30(5) + 10(5)^2 - \frac{10(5)^3}{3}$$

$$= 400 - \frac{1250}{3} \left(= -\frac{50}{3}\right)$$ ***A1***

total displacement is negative **OR** total displacement is not zero, (so the object does return to its initial position) ***R1***

METHOD 2

attempt to find displacement after change in direction by setting up an integral of $v(t)$ between $t = 3$ and $t = 5$ ***(M1)***

$$\int_3^5 (30 + 20t - 10t^2)\ \mathrm{d}\ t$$

$$= \left[30t + 10t^2 - \frac{10t^3}{3}\right]_3^5$$ ***(A1)***

$$= \left(30(5) + 10(5)^2 - \frac{10(5)^3}{3}\right) - \left(30(3) + 10(3)^2 - \frac{10(3)^3}{3}\right) = \left(400 - \frac{1250}{3}\right) - 90$$

$= -\frac{50}{3} - 90\left(= -\frac{320}{3}\right)$ ***A1***

$\frac{320}{3} > 90$ compares to distance before change of direction, (so the object does return to its initial position) ***R1***

[4 marks]

6. [Maximum mark: 7]

A particle P moves in a straight line. The velocity $v\,\mathrm{m\,s^{-1}}$ of P, at time t seconds is given by $v(t) = \mathrm{e}^{-\sin t} \cos(2t)$, for $0 \le t \le 5$.

(a) Find the maximum speed of P. [2]

Markscheme

recognition that maximum speed occurs when $|v|$ is greatest ***(M1)***

one correct coordinate for minimum $(4.71238\ldots,\ -2.71828\ldots)$

$2.72\ (= \mathrm{e})\ \left(\mathrm{ms^{-1}}\right)$ ***A1***

Note: Award ***(M1)A0*** for the answer 1, from working with degrees.

[2 marks]

(b) Find the total distance travelled by P. [2]

Markscheme

substitution of limits into correct formula ***(M1)***

$\int_0^5 |v| \, \mathrm{d}\, t$

$3.84591\ldots$

3.85 (metres) ***A1***

[2 marks]

(c) Find the acceleration when P changes direction for the **second** time. [3]

Markscheme

$v = 0$ (seen anywhere) ***(M1)***

$t = 2.35619\ldots \left(= \frac{3\pi}{4}\right)$ ***(A1)***

$a = 0.986137\ldots$

$a = 0.986 \left(= 2\mathrm{e}^{-\frac{1}{\sqrt{2}}}\right) \left(\mathrm{ms}^{-2}\right)$ ***A1***

[3 marks]

7. [Maximum mark: 14]

Two athletes, Fiona and Lucy, compete in a 200 metres race along a straight track.

Fiona's velocity, in $\mathrm{m\,s}^{-1}$, during the race can be modelled by $v(t) = \frac{8.14t}{\sqrt{t^2+0.2}}$, where $t \geq 0$. Time, t, is measured in seconds from when the race starts.

(a.i) Write down the value of $v(1)$. [1]

Markscheme

$7.43076\ldots$

$v(1) = 7.43 \left(\mathrm{ms}^{-1}\right)$ ***A1***

[1 mark]

(a.ii) Find the time when Fiona's velocity is $5\,\mathrm{m\,s}^{-1}$. [2]

Markscheme

equating $v(t)$ and 5 ***(M1)***

$0.348114...$

0.348 (seconds) ***A1***

[2 marks]

(b) Find the time when Fiona's acceleration is $4\,\mathrm{m\,s^{-2}}$. [2]

Markscheme

recognizing $a = v\prime(t)$ ***(M1)***

$0.590930...$

0.591 (seconds) ***A1***

[2 marks]

(c.i) Write down the limit of $v(t)$ as t approaches infinity. [2]

Markscheme

considering large values of t ***(M1)***

$\left(\lim_{t\to\infty}(v(t)) =\right) 8.14\,(\mathrm{m\,s^{-1}})$ ***A1***

[2 marks]

(c.ii) State a reason why the value in part (c)(i) is not valid in the context of this question [1]

Markscheme

EITHER

the race lasts a finite time (e.g. it ends after (Fiona) crosses the line) ***R1***

OR

the graph approaches the limiting value but Fiona will never attain that speed ***R1***

OR

Fiona cannot maintain that speed for a long period of time ***R1***

Note: Award ***R1*** for a valid reason that does not contradict their answer to part (c)(i).

[1 mark]

Lucy's velocity, in $\mathrm{m\,s^{-1}}$, during the race can be modelled by $w(t) = \frac{8t}{\sqrt{t^2+0.3}}$, where $t \geq 0$.

Fiona completes the race and crosses the finishing line in front of Lucy.

(d) Find the distance Lucy is from the finishing line when Fiona completes the 200 metres. [6]

Markscheme

Fiona takes t_f seconds to travel $200\,\mathrm{m}$

EITHER

recognizing distance travelled by either athlete in the first t seconds is

$\int_0^t v(t)\ \mathrm{d}\,t$ **OR** $\int_0^t w(t)\ \mathrm{d}\,t$ (seen anywhere) ***(M1)***

equating distance travelled by Fiona to $200\,(\mathrm{m})$ ***(M1)***

$$\int_0^{t_f} v(t)\ \mathrm{d}\,t = 200$$

OR

attempt to integrate $v(t)$ ***(M1)***

$$s(t) = 8.14\sqrt{t^2+0.2} - 3.64031\ldots$$

equating 200 to *their* $s(t)$ (must include a constant of integration) ***(M1)***

$$8.14\sqrt{t^2+0.2} - 3.64031\ldots = 200$$

THEN

$t_f = 25.0132\ldots$ (accept 25) ***(A1)***

recognition that a definite integral of $w(t)$ with 0 and *their* t_f is required ***(M1)***

(Lucy's distance $=$) $\int_0^{25.0132\ldots} w(t)\ \mathrm{d}\,t\ \ (= 195.772\ldots)$

distance from finishing line $= 200 -$ *their* Lucy's distance ***(M1)***

$4.22788\ldots$

$4.23\,(\mathrm{m})$ ***A1***

[6 marks]

8. [Maximum mark: 16]

In this question all values of x and t are in radians.

Consider the function $f(x) = 3\sin(4\pi x)$.

(a.i) Write down the amplitude of the graph of f. [1]

Markscheme

3 ***A1***

[1 mark]

(a.ii) Find the period of f. [2]

Markscheme

attempt to find period ***(M1)***

$\frac{2\pi}{b}$ or $\frac{2\pi}{4\pi}$

period $= \frac{1}{2}$ ***A1***

[2 marks]

Consider a second function $g(x) = -4\cos(4\pi x)$.

The sum of these functions can be expressed in the form $f(x) + g(x) = a\cos(b(x - c))$, where $a,\ b,\ c > 0$.

(b) By considering the graph of $y = f(x) + g(x)$, determine

(b.i) the value of a; [2]

Markscheme

evidence of considering the graph of $3\sin(4\pi x) - 4\cos(4\pi x)$ (seen in i, ii,or iii) ***(M1)***

$a = 5$ ***A1***

[2 marks]

(b.ii) the value of b; [1]

Markscheme

$b = 4\pi$ ***A1***

[1 mark]

(b.iii) the smallest possible value of c. [1]

Markscheme

$0.198792\ldots$

$c = 0.199$ ***A1***

[1 mark]

A car is travelling along a straight residential street with speed bumps placed at regular intervals on the road to encourage safer driving. The car travels at a minimum velocity when passing over speed bumps and reaches a maximum velocity in between speed bumps.

Its velocity, in $\mathrm{m\,s^{-1}}$, can be modelled by the function $v(t) = -3.5\cos\left(\frac{\pi}{14}(t-5)\right) + 9$, where t is measured in seconds.

(c) Find the time at which the car first reaches its maximum velocity. [1]

Markscheme

19 (seconds) ***A1***

[1 mark]

(d) Find the number of speed bumps the car passes over in the first two minutes of motion. [1]

Markscheme

5 (speed bumps) ***A1***

[1 mark]

(e.i) Find $v\prime(t)$. [2]

Markscheme

attempt to use chain rule (multiplication by $\frac{\pi}{14}$) ***(M1)***

$(3.5)\left(\frac{\pi}{14}\right)\sin\left(\frac{\pi}{14}(t-5)\right)$ OR $0.785398\ldots$

$v\prime(t) = 0.785 \sin\left(\frac{\pi}{14}(t-5)\right) \ \left(= \frac{\pi}{4}\sin\left(\frac{\pi}{14}(t-5)\right)\right)$ ***A1***

[2 marks]

(e.ii) Hence, or otherwise, write down the maximum acceleration of the car.

[2]

Markscheme

recognition that $v\prime = a$ ***(M1)***

$0.785\ (\mathrm{ms}^{-2})\ \left(= \frac{\pi}{4}\right)$ ***A1***

[2 marks]

(f) Find the distance, in metres, between consecutive speed bumps. [3]

Markscheme

recognition that a definite integral of the velocity function is needed ***(M1)***

using a correct set of limits (any limits which differ by 28 seconds) ***(A1)***

$\int_5^{33} v(t)\ \mathrm{d}\ t \ \left(= \int_k^{k+28} |v(t)|\ \mathrm{d}\ t\right)$

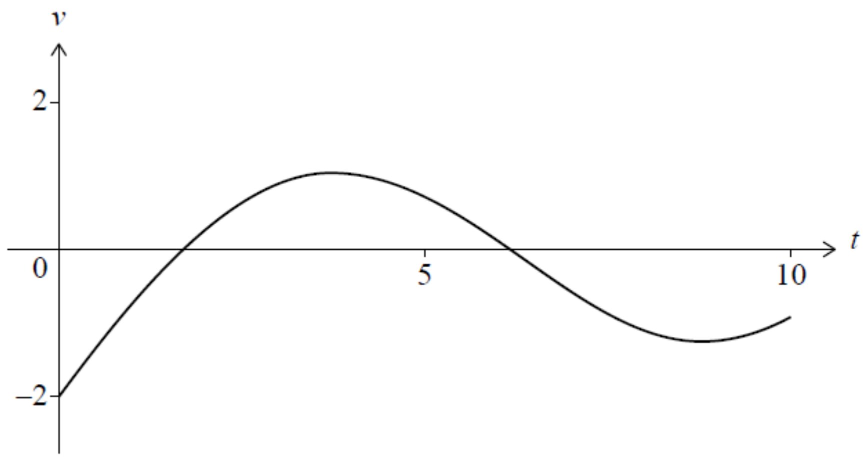
$252\ \ (\mathrm{m})$ (exact) ***A1***

[3 marks]

9. [Maximum mark: 6]

A particle moves in a straight line such that it passes through a fixed point O at time $t = 0$, where t represents time measured in seconds after passing O. For $0 \le t \le 10$ its velocity, v metres per second, is given by $v = 2\sin(0.5t) + 0.3t - 2$.

The graph of v is shown in the following diagram.



(a) Find the smallest value of t when the particle changes direction. [2]

Markscheme

recognition that velocity is zero *(M1)*

$v = 2\sin(0.5t) + 0.3t - 2 = 0$

$t = 1.68694\ldots$

$t = 1.69$ ***A1***

[2 marks]

The displacement of the particle is measured in metres from O.

(b) Find the range of values of t for which the velocity is positive. [2]

Markscheme

$1.68694\ldots < t < 6.11857\ldots$

$1.69 < t < 6.12$ ***A1A1***

Note: Award ***A1*** for both values, ***A1*** for correct inequalities.

[2 marks]

(c) Find the displacement of the particle relative to O when $t = 10$. [2]

Markscheme

attempt to substitute into the total displacement formula (condone missing or incorrect limits, and absence of $\mathrm{d}\,t$) *(M1)*

$\int_0^{10}(2\sin(0.5t)+0.3t-2)\,\mathrm{d}t$ OR $\int_0^{10} v(t)\,\mathrm{d}t$

$=-2.13464\ldots$

$=-2.13\,(\mathrm{m})$ ***A1***

Note: Award ***(M1)A0*** if -2.13 is followed by 2.13.

[2 marks]

10. [Maximum mark: 6]

A particle moves in a straight line such that its velocity, $v\ \mathrm{ms}^{-1}$, at time t seconds is given by $v(t)=1+\mathrm{e}^{-t}-\mathrm{e}^{-\sin 2t}$ for $0\le t\le 2$.

(a) Find the velocity of the particle at $t=2$. [1]

Markscheme

$v=-0.996114\ldots$

$v=-0.996$ (ms^{-1}) ***A1***

[1 mark]

(b) Find the maximum velocity of the particle. [2]

Markscheme

considers $v\prime(t)=0$ ***(M1)***

$t=0.405833\ldots$

$v_{\max}=1.18230\ldots$

$v_{\max}=1.18$ (ms^{-1}) ***A1***

[2 marks]

(c) Find the acceleration of the particle at the instant it changes direction. [3]

Markscheme

recognizes that the particle changes direction when $v=0$ ***(M1)***

Note: Award *(M1)* for $t = 1.65840\ldots$ seen.

finds acceleration for their value of t for which $v(t) = 0$ *(M1)*

$v\prime(1.65840\ldots)$

$a = -2.53487\ldots$

$a = -2.53$ (ms^{-2}) *A1*

[3 marks]

11. [Maximum mark: 5]

A particle moves along a straight line. Its displacement, s metres, from a fixed point O after time t seconds is given by $s(t) = 5.2 \sin\left(\sqrt{4t+6}\right)$, where $0 \leq t \leq 10.$

The particle first comes to rest after q seconds.

(a) Find the value of q. [2]

Markscheme

recognizing at rest when $\frac{\mathrm{d}s}{\mathrm{d}t} = 0$ OR s is a minimum *(M1)*

$q = 4.05165\ldots$

$= 4.05$ *A1*

[2 marks]

(b) Find the total distance that the particle travels in the first q seconds. [3]

Markscheme

METHOD 1

recognizing that integral of $v(t)$ is required *(M1)*

$\int_0^{4.05\ldots} |v(t)|\ \mathrm{d}\ t$ OR $\int_0^{4.05\ldots} \left|\frac{\mathrm{d}}{\mathrm{d}t} s(t)\right|\ \mathrm{d}\ t$ OR $\left|\int_0^{4.05\ldots} v(t)\ \mathrm{d}\ t\right|$ OR $-\int_0^{4.05\ldots} v(t)\ \mathrm{d}\ t$ *(A1)*

Note: Only accept $\left|\int_0^q v(t)\ \mathrm{d}\ t\right|$ if their value of q does not result in the particle changing direction in the first q seconds.

$= 8.51841\ldots$

$= 8.52$ (m) *A1*

METHOD 2

recognition that total distance travelled is the difference between the initial displacement and the displacement at minimum *(M1)*

initial displacement is $3.31841\ldots$ AND at minimum is -5.2 *(A1)*

total distance travelled $= 3.31841\ldots - (-5.2)$

$= 8.51841\ldots$

$= 8.52$ (m) *A1*

[3 marks]

12. [Maximum mark: 16]

A farmer is growing a field of wheat plants. The height, H cm, of each plant can be modelled by a normal distribution with mean μ and standard deviation σ.

It is known that $\mathrm{P}(H < 94.6) = 0.288$ and $\mathrm{P}(H > 98.1) = 0.434$.

(a) Find the probability that the height of a randomly selected plant is between 94.6 cm and 98.1 cm. [2]

Markscheme

recognizing probabilities sum to 1 *(M1)*

$0.288 + \mathrm{P}(94.6 < X < 98.1) + 0.434 = 1$

$\mathrm{P}(94.6 < X < 98.1) = 0.278$ *A1*

[2 marks]

(b) Find the value of μ and the value of σ. [5]

Markscheme

METHOD 1

recognizing the need to use inverse normal with $0.288,\ (1 - 0.434)$ or 0.434 *(M1)*

$\mu + \mathrm{invNorm}(0.288)\sigma = 94.6,\ \mu + \mathrm{invNorm}(1 - 0.434)\sigma = 98.1$ (or equivalent) *(A1)(A1)* attempt to solve their equations in two variables using the GDC (that involve either z-values or 'invNorm' rather than probabilities) ***(M1)***

$\mu = 97.2981\ldots, \sigma = 4.82468\ldots$

$\mu = 97.3, \quad \sigma = 4.82$ ***A1***

Note: Condone use of different variables throughout, but do not award the final ***A1*** if they do not clearly identify which variable is their mean and standard deviation.

METHOD 2

use of inverse normal to find at least one z-score for $\mathrm{P}(Z < z) = 0.288$ or $\mathrm{P}(Z < z) = 1 - 0.434$ ***(M1)***

$z_1 = -0.559236\ldots$ OR $z_2 = 0.166199\ldots$

$\frac{94.6-\mu}{\sigma} = -0.559236\ldots, \frac{98.1-\mu}{\sigma} = 0.166199\ldots$ (or equivalent) ***(A1)(A1)***

attempt to solve their equations (that involve z-values rather than probabilities) ***(M1)***

$\mu = 97.2981\ldots, \sigma = 4.82468\ldots$

$\mu = 97.3, \quad \sigma = 4.82$ ***A1***

[5 marks]

The farmer measures 100 randomly selected plants. Any plant with a height greater than $98.1\ \mathrm{cm}$ is considered ready to harvest. Heights of plants are independent of each other.

(c.i) Find the probability that exactly 34 plants are ready to harvest. [2]

Markscheme

recognition of Binomial distribution **(M1)**

$X \sim \mathrm{B}(100, 0.434)$

$\mathrm{P}(X = 34) = 0.0133198\ldots$

$= 0.0133$ ***A1***

[2 marks]

(c.ii) Given that fewer than 49 plants are ready to harvest, find the probability that exactly 34 plants are ready to harvest. [4]

Markscheme

$\mathrm{P}(X < 49) = 0.848218\ldots$ (seen anywhere) *(A1)*

recognition of conditional probability *(M1)*

Note: recognition must be shown in context, either in symbols e.g. $\mathrm{P}(X = 34 \mid X < 49)$, or in words e.g. $\mathrm{P}(34 \text{ plants} \mid \text{less than } 49 \text{ plants})$, not only as $\mathrm{P}(A \mid B)$.

$(\mathrm{P}(X = 34 \mid X < 49) =) \frac{\mathrm{P}(X=34)}{\mathrm{P}(X<49)}$ OR $\frac{\mathrm{P}(X=34)}{\mathrm{P}(X \le 48)}$ $\left(= \frac{0.0133198\ldots}{0.848218\ldots}\right)$ *(A1)*

$= 0.0157033\ldots$

$\mathrm{P}(X = 34 \mid X < 49) = 0.0157$ *A1*

[4 marks]

In another field, the farmer is growing the same variety of wheat, but is using a different fertilizer. The heights of these plants, F cm, are normally distributed with mean 98.6 and standard deviation d. The farmer finds the interquartile range to be 4.82 cm.

(d) Find the value of d. [3]

Markscheme

$\mathrm{Q}_1 = 96.19$ OR $\mathrm{Q}_3 = 101.01$ (may be seen on a labelled diagram with areas indicated) *(A1)*

$\mathrm{P}(96.19 < F < 101.01) = 0.5$ OR $\mathrm{P}(F < 96.19) = 0.25$ OR $\mathrm{P}(F < 101.01) = 0.75$ (or equivalent)

EITHER

attempt to find d using graph or table *(M1)*

OR

$1 - 2\mathrm{P}\left(Z < -\frac{2.41}{d}\right) = 0.5$ OR $\mathrm{P}\left(Z < -\frac{2.41}{d}\right) = 0.25$ OR $\mathrm{P}\left(Z < \frac{2.41}{d}\right) = 0.75$ OR $\mathrm{P}\left(-\frac{2.41}{d} < Z < \frac{2.41}{d}\right) = 0.5$ (or equivalent) *(M1)*

$-\frac{2.41}{d} = -0.674489\ldots$ OR $\frac{2.41}{d} = 0.674489\ldots$

THEN

$3.57307\ldots$

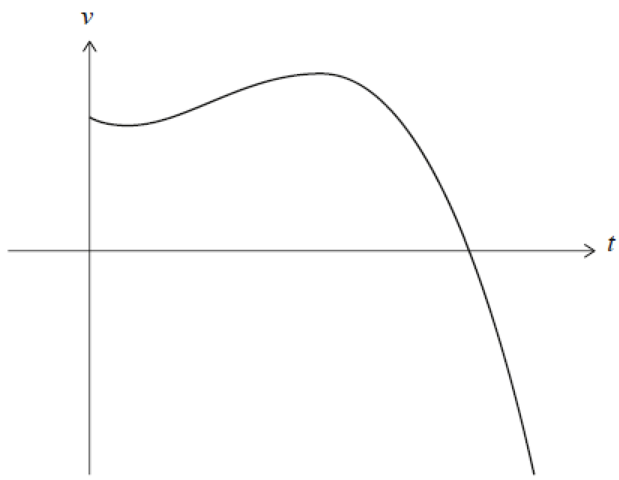
$d = 3.57$ *A1*

[3 marks]

13. [Maximum mark: 17]

An object moves along a straight line. Its velocity, $v\,\mathrm{m\,s^{-1}}$, at time t seconds is given by $v(t) = -t^3 + \frac{7}{2}t^2 - 2t + 6$, for $0 \le t \le 4$. The object first comes to rest at $t = k$.

The graph of v is shown in the following diagram.



At $t = 0$, the object is at the origin.

(a) Find the displacement of the object from the origin at $t = 1$. [5]

Markscheme

attempt to integrate v (integration of at least one term) ***(M1)***

$(s(t) =) -\frac{1}{4}t^4 + \frac{7}{6}t^3 - t^2 + 6t\ (+C)$ ***A2***

Note: Award ***A1*** for at least two correct terms.

substitution of $t = 1$ into their integrated expression ***(M1)***

displacement $= 5\frac{11}{12} \left(= \frac{71}{12}\right) (\mathrm{m})$ ***A1***

[5 marks]

(b) Find an expression for the acceleration of the object. [2]

Markscheme

attempt to differentiate v (differentiation of at least one term) ***(M1)***

$a(t) = -3t^2 + 7t - 2$ ***A1***

[2 marks]

(c) Hence, find the greatest speed reached by the object before it comes to rest. [5]

Markscheme

setting their $v\prime(t) = 0$ ***(M1)***

$-3t^2 + 7t - 2 = 0$

valid attempt to solve quadratic ***(M1)***

$(3t - 1)(t - 2) = 0$ OR $\frac{-7 \pm \sqrt{49 - 4(-3)(-2)}}{-6}$

$t = \frac{1}{3},\ 2$ ($t = \frac{1}{3}$ may be omitted) ***(A1)***

substitute their largest positive t-value into $v(t)$ ***(M1)***

greatest speed is $8\ (\mathrm{ms}^{-1})$ ***A1***

[5 marks]

(d) Find the greatest speed reached by the object for $0 \le t \le 4$. [2]

Markscheme

attempt to check other boundary value at $t = 4$ ***(M1)***

$v(4) = -64 + 56 - 8 + 6\ (= -10)$

greatest speed is $10\ \mathrm{ms}^{-1}$ ***A1***

[2 marks]

(e) Write down an expression that represents the distance travelled by the object while its speed is increasing. Do not evaluate the expression. [3]

Markscheme

identifying correct intervals where speed increases (may be seen in integral) ***(A1)(A1)***

$t = \frac{1}{3}$ to $t = 2$ and $t = k$ to $t = 4$

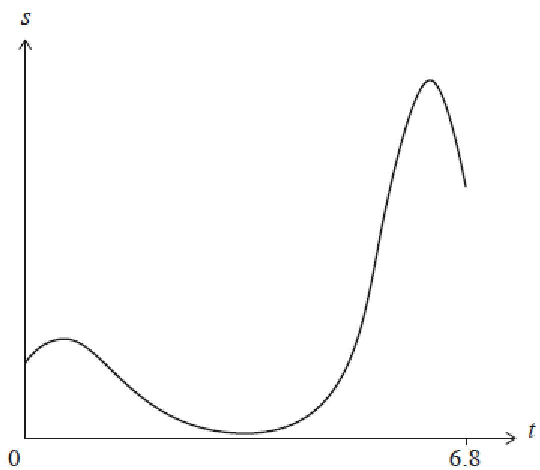
$\int_{\frac{1}{3}}^{2} v\left(t\right)\mathrm{d}t + \int_{k}^{4}|v(t)|\mathrm{d}t$ OR $\int_{\frac{1}{3}}^{2} v\mathrm{d}t + \left|\int_{\mathrm{k}}^{4} \mathrm{vd}\,t\right|$ OR $\int_{\frac{1}{3}}^{2} v\left(t\right)\mathrm{d}t - \int_{k}^{4} v\left(t\right)\mathrm{d}t$ ***A1***

Note: Condone missing $\mathrm{d}t$.

[3 marks]

14. [Maximum mark: 16]

A particle moves in a straight line. Its displacement, s metres, from a fixed point P at time t seconds is given by $s\left(t\right) = 3(t+2)^{\cos t}$, for $0 \le t \le 6.8$, as shown in the following graph.



(a) Find the particle's initial displacement from the point P. [2]

Markscheme

initial displacement is $s(0)$ ***(M1)***

$6\,(\mathrm{m})$ ***A1***

[2 marks]

(b) Find the particle's velocity when $t = 2$. [2]

Markscheme

velocity is s' ***(M1)***

-2.29920

-2.30 (m/s) *A1*

[2 marks]

(c) Determine the intervals of time when the particle is moving away from the point P. [5]

Markscheme

attempting to find t when the particle changes direction ***(M1)***

$t = 0.433007\ldots$ AND $3.25575\ldots$ AND $6.33965\ldots$ (may be seen on a graph) ***(A1)***

particle travels away from P when $v > 0$ OR when $s' > 0$ ***(M1)***

$0 \leq t < 0.433007\ldots,\ 3.25575\ldots < t < 6.33965$

$0 \leq t < 0.433,\ 3.26 < t < 6.34$ ***A1A1***

[5 marks]

The acceleration of the particle is zero when $t = b$ and $t = c$, where $b < c$.

(d) Find the value of b and the value of c. [4]

Markscheme

recognizing that acceleration is $a(t) = v'(t)$ OR $a(t) = s''(t)$ ***(M1)***

attempting to find max/min on graph of velocity OR finding zeros on graph of acceleration ***(M1)***

$b = 1.23140\ldots,\ c = 5.68959\ldots$

$b = 1.23,\ c = 5.69$ ***A1A1***

[4 marks]

(e) Find the total distance travelled by the particle for $b \leq t \leq c$. [3]

Markscheme

METHOD 1 (using integral of velocity)

correct integral (accept absence of $\mathrm{d}t$) ***(A1)***

$\int_{1.23140...}^{5.68959...} |v(t)|\ \mathrm{d}\,t$ OR $\int_b^c |s\prime(t)|\ \mathrm{d}\,t$ OR $-\int_{1.23140...}^{3.25575...} v(t)\ \mathrm{d}\,t + \int_{3.25575...}^{5.68959...} v(t)\ \mathrm{d}\,t$ OR $3.8560 + 15.696$

$19.5525\ldots$

total distance $= 19.6\ (\mathrm{m})$ ***A2***

METHOD 2 (using differences in displacement)

finding displacement at b, c **and** local min on displacement graph ***(A1)***

$(b,\ 4.43306),\ (c,\ 16.2734),\ (3.25575,\ 0.577001)$ OR $4.43306,\ 0.577001,\ 16.2734$

correct approach ***(A1)***

$(4.43306 - 0.577001) + (16.2734 - 0.577001)$ OR towards $\mathrm{P}\ 3.85606 +$ away from $\mathrm{P}\ 15.696$

$19.5525\ldots$

total distance $= 19.6\ (\mathrm{m})$ ***A1***

[3 marks]

15. [Maximum mark: 7]

A particle moves along a straight line so that its velocity, $v\,\mathrm{m\,s}^{-1}$, after t seconds is given by $v(t) = \mathrm{e}^{\sin t} + 4\sin t$ for $0 \le t \le 6$.

(a) Find the value of t when the particle is at rest. [2]

Markscheme

recognizing at rest $v = 0$ ***(M1)***

$t = 3.34692\ldots$

$t = 3.35$ (seconds) ***A1***

Note: Award ***(M1)A0*** for additional solutions to $v = 0$ eg $t = -0.205$ or $t = 6.08$.

[2 marks]

(b) Find the acceleration of the particle when it changes direction. [3]

Markscheme

recognizing particle changes direction when $v = 0$ OR when $t = 3.34692\ldots$ ***(M1)***

$a = -4.71439\ldots$

$a = -4.71\ \left(\mathrm{ms}^{-2}\right)$ ***A2***

[3 marks]

(c) Find the total distance travelled by the particle. [2]

Markscheme

distance travelled $= \int_0^6 |v|\ \mathrm{d}\ t$ OR

$\int_0^{3.34\ldots} \left(\mathrm{e}^{\sin(t)} + 4\ \sin(t)\right)\ \mathrm{d}\ t - \int_{3.34\ldots}^{6} \left(\mathrm{e}^{\sin(t)} + 4\ \sin(t)\right)\ \mathrm{d}\ t\ \ (= 14.3104\ldots + 6.44300\ldots)$ ***(A1)***

$= 20.7534\ldots$

$= 20.8$ (metres) ***A1***

[2 marks]

16. [Maximum mark: 7]

A particle moves in a straight line such that its velocity, $v\,\mathrm{m\,s}^{-1}$, at time t seconds is given by $v = \frac{\left(t^2+1\right)\cos t}{4},\ 0 \le t \le 3.$

(a) Determine when the particle changes its direction of motion. [2]

Markscheme

recognises the need to find the value of t when $v = 0$ ***(M1)***

$t = 1.57079\ldots\ \left(= \frac{\pi}{2}\right)$

$t = 1.57\ \left(= \frac{\pi}{2}\right)$ (s) ***A1***

[2 marks]

(b) Find the times when the particle's acceleration is $-1.9\mathrm{ms}^{-2}$. [3]

Markscheme

recognises that $a(t) = v\prime(t)$ ***(M1)***

$t_1 = 2.26277\ldots,\ t_2 = 2.95736\ldots$

$t_1 = 2.26,\ t_2 = 2.96$ (s) ***A1A1***

Note: Award ***M1A1A0*** if the two correct answers are given with additional values outside $0 \leq t \leq 3$.

[3 marks]

(c) Find the particle's acceleration when its speed is at its greatest. [2]

Markscheme

speed is greatest at $t = 3$ ***(A1)***

$a = -1.83778\ldots$

$a = -1.84\ (\mathrm{ms}^{-2})$ ***A1***

[2 marks]

17. [Maximum mark: 16]

A particle P moves along the x-axis. The velocity of P is $v\,\mathrm{ms}^{-1}$ at time t seconds, where $v(t) = 4 + 4t - 3t^2$ for $0 \leq t \leq 3$. When $t = 0,\ P$ is at the origin O.

(a.i) Find the value of t when P reaches its maximum velocity. [2]

Markscheme

valid approach to find turning point ($v\prime = 0,\ -\frac{b}{2a}$, average of roots) ***(M1)***

$4 - 6t = 0$ OR $-\frac{4}{2(-3)}$ OR $\frac{-\frac{2}{3}+2}{2}$

$t = \frac{2}{3}$ (s) ***A1***

[2 marks]

(a.ii) Show that the distance of P from O at this time is $\frac{88}{27}$ metres. [5]

Markscheme

attempt to integrate v ***(M1)***

$\int v \mathrm{~d} t=\int\left(4+4t-3t^{2}\right) \mathrm{~d} t=4t+2t^{2}-t^{3}(+c)$ ***A1A1***

Note: Award ***A1*** for $4t+2t^{2}$, ***A1*** for $-t^{3}$.

attempt to substitute their t into their solution for the integral ***(M1)***

distance $=4\left(\frac{2}{3}\right)+2\left(\frac{2}{3}\right)^{2}-\left(\frac{2}{3}\right)^{3}$

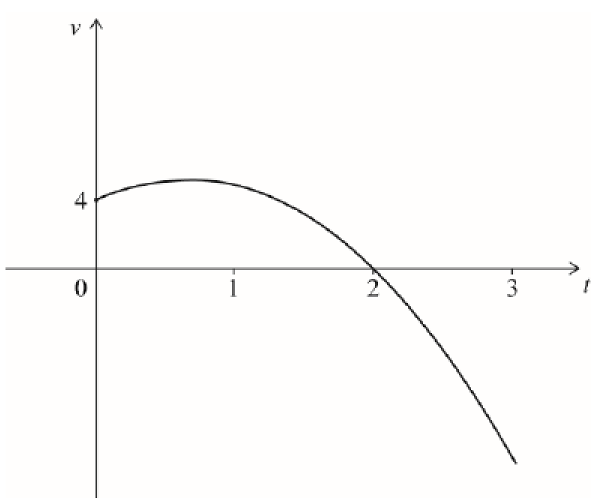
$=\frac{8}{3}+\frac{8}{9}-\frac{8}{27}$ (or equivalent) ***A1***

$=\frac{88}{27}$ (m) ***AG***

[5 marks]

(b) Sketch a graph of v against t, clearly showing any points of intersection with the axes. [4]

Markscheme



valid approach to solve $4+4t-3t^{2}=0$ (may be seen in part (a)) ***(M1)***

$(2 - t)(2 + 3t)$ OR $\frac{-4 \pm \sqrt{16+48}}{-6}$

correct x- intercept on the graph at $t = 2$ ***A1***

Note: The following two ***A*** marks may only be awarded if the shape is a concave down parabola. These two marks are independent of each other and the ***(M1)***.

correct domain from 0 to 3 starting at $(0, 4)$ ***A1***

Note: The 3 must be clearly indicated.

vertex in approximately correct place for $t = \frac{2}{3}$ and $v > 4$ ***A1***

[4 marks]

(c) Find the total distance travelled by P. [5]

Markscheme

recognising to integrate between 0 and 2, or 2 and 3 OR $\int_0^3 \left|4 + 4t - 3t^2\right| \mathrm{d}\, t$ ***(M1)***

$\int_0^2 \left(4 + 4t - 3t^2\right) \mathrm{d}\, t$

$= 8$ ***A1***

$\int_2^3 \left(4 + 4t - 3t^2\right) \mathrm{d}\, t$

$= -5$ ***A1***

valid approach to sum the two areas (seen anywhere) ***(M1)***

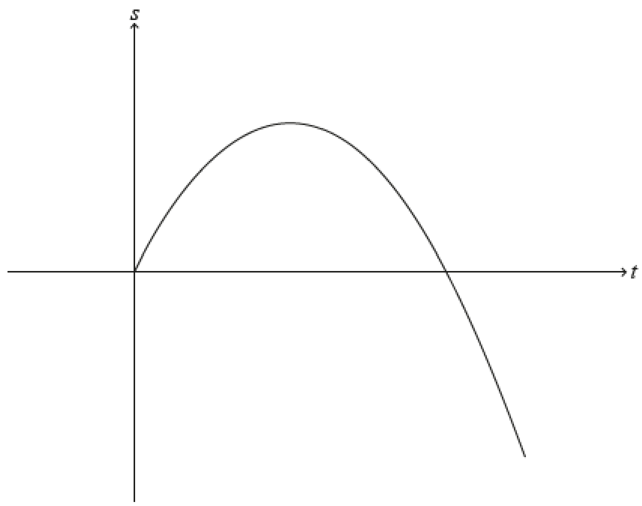
$\int_0^2 v \mathrm{d}\, t - \int_2^3 v \mathrm{d}\, t$ OR $\int_0^2 v \mathrm{d}\, t + \left|\int_2^3 v \mathrm{d}\, t\right|$

total distance travelled $= 13$ (m) ***A1***

[5 marks]

18. [Maximum mark: 14]

Particle A travels in a straight line such that its displacement, s metres, from a fixed origin after t seconds is given by $s(t) = 8t - t^2$, for $0 \leq t \leq 10$, as shown in the following diagram.



Particle A starts at the origin and passes through the origin again when $t = p$.

(a) Find the value of p. [2]

Markscheme

setting $s(t) = 0$ ***(M1)***

$8t - t^2 = 0$

$t(8 - t) = 0$

$p = 8$ (accept $t = 8,\ (8, 0)$) ***A1***

Note: Award ***A0*** if the candidate's final answer includes additional solutions (such as $p = 0,\ 8$).

[2 marks]

Particle A changes direction when $t = q$.

(b.i) Find the value of q. [2]

Markscheme

recognition that when particle changes direction $v = 0$ OR local maximum on graph of s OR vertex of parabola ***(M1)***

$q = 4$ (accept $t = 4$) *A1*

[2 marks]

(b.ii) Find the displacement of particle A from the origin when $t = q$. [2]

Markscheme

substituting their value of q into $s(t)$ OR integrating $v(t)$ from $t = 0$ to $t = 4$ *(M1)*

$\text{displacement} = 16\,(\text{m})$ *A1*

[2 marks]

(c) Find the distance of particle A from the origin when $t = 10$. [2]

Markscheme

$s(10) = -20$ OR $\text{distance} = |s(t)|$ OR integrating $v(t)$ from $t = 0$ to $t = 10$ *(M1)*

$\text{distance} = 20\,(\text{m})$ *A1*

[2 marks]

The total distance travelled by particle A is given by d.

(d) Find the value of d. [2]

Markscheme

16 forward $+$ 36 backward OR $16 + 16 + 20$ OR $\int_0^{10} |v(t)|\,\mathrm{d}\,t$ *(M1)*

$d = 52\,(\text{m})$ *A1*

[2 marks]

(e) A second particle, particle B, travels along the same straight line such that its velocity is given by $v(t) = 14 - 2t$, for $t \geq 0$.

When $t = k$, the distance travelled by particle B is equal to d.

Find the value of k.

[4]

Markscheme

METHOD 1

graphical method with triangles on $v(t)$ graph ***M1***

$49 + \left(\frac{x(2x)}{2}\right)$ ***(A1)***

$49 + x^2 = 52,\ x = \sqrt{3}$ ***(A1)***

$k = 7 + \sqrt{3}$ ***A1***

METHOD 2

recognition that distance $= \int |v(t)|\ \mathrm{d}\ t$ ***M1***

$\int_0^7 (14 - 2t)\ \mathrm{d}\ t + \int_7^k (2t - 14)\ \mathrm{d}\ t$

$\left[14t - t^2\right]_0^7 + \left[t^2 - 14t\right]_7^k$ ***(A1)***

$14(7) - 7^2 + \left(\left(k^2 - 14k\right) - \left(7^2 - 14(7)\right)\right) = 52$ ***(A1)***

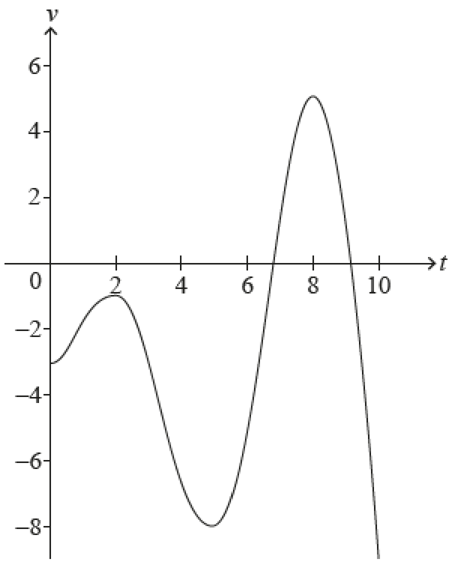
$k = 7 + \sqrt{3}$ ***A1***

[4 marks]

19. [Maximum mark: 6]

A particle moves in a straight line. The velocity, $v\,\mathrm{m\,s}^{-1}$, of the particle at time t seconds is given by $v(t) = t\,\sin\,t - 3$, for $0 \le t \le 10$.

The following diagram shows the graph of v.



(a) Find the smallest value of t for which the particle is at rest. [2]

Markscheme

recognising $v = 0$ *(M1)*

$t = 6.74416\ldots$

$= 6.74$ (sec) ***A1***

Note: Do not award ***A1*** if additional values are given.

[2 marks]

(b) Find the total distance travelled by the particle. [2]

Markscheme

$\int_0^{10} |v(t)|\ \mathrm{d}\ t$ OR $-\int_0^{6.74416\ldots} v(t)\ \mathrm{d}\ t + \int_{6.74416\ldots}^{9.08837\ldots} v(t)\ \mathrm{d}\ t - \int_{9.08837\ldots}^{10} v(t)\ \mathrm{d}\ t$ *(A1)*

$= 37.0968\ldots$

$= 37.1\ (\mathrm{m})$ ***A1***

[2 marks]

(c) Find the acceleration of the particle when $t = 7$. [2]

Markscheme

recognizing acceleration at $t = 7$ is given by $v\prime(7)$ ***(M1)***

acceleration $= 5.93430\ldots$

$= 5.93\ \left(\mathrm{ms}^{-2}\right)$ ***A1***

[2 marks]

20. [Maximum mark: 7]
In this question, all lengths are in metres and time is in seconds.

Consider two particles, P_1 and P_2, which start to move at the same time.

Particle P_1 moves in a straight line such that its displacement from a fixed-point is given by $s(t) = 10 - \frac{7}{4}t^2$, for $t \geq 0$.

(a) Find an expression for the velocity of P_1 at time t. [2]

Markscheme

* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

recognizing velocity is derivative of displacement ***(M1)***

eg $v = \frac{\mathrm{d}s}{\mathrm{d}t}$, $\frac{\mathrm{d}}{\mathrm{d}t}\left(10 - \frac{7}{4}t^2\right)$

velocity$= -\frac{14}{4}t \quad \left(= -\frac{7}{2}t\right)$ ***A1 N2***

[2 marks]

(b) Particle P_2 also moves in a straight line. The position of P_2 is given by $\boldsymbol{r} = \begin{pmatrix} -1 \\ 6 \end{pmatrix} + t\begin{pmatrix} 4 \\ -3 \end{pmatrix}$.

The speed of P_1 is greater than the speed of P_2 when $t > q$.

Find the value of q. [5]

Markscheme

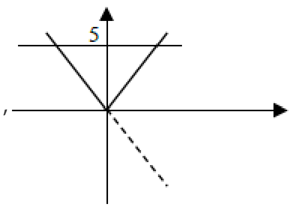
valid approach to find speed of P_2 ***(M1)***

eg $\left|\begin{pmatrix} 4 \\ -3 \end{pmatrix}\right|$, $\sqrt{4^2 + (-3)^2}$, velocity$= \sqrt{4^2 + (-3)^2}$

correct speed ***(A1)***

eg $5\,\mathrm{m\,s^{-1}}$

recognizing relationship between speed and velocity (may be seen in inequality/equation) ***R1***

eg $\left|-\frac{7}{2}t\right|$, speed = | velocity |, graph of P_1 speed, 

P_1 speed $= \frac{7}{2}t$, P_2 velocity $= -5$

correct inequality or equation that compares speed or velocity (accept any variable for q) ***A1***

eg $\left|-\frac{7}{2}t\right| > 5$, $-\frac{7}{2}q < -5$, $\frac{7}{2}q > 5$, $\frac{7}{2}q = 5$

$q = \frac{10}{7}$ (seconds) (accept $t > \frac{10}{7}$, do not accept $t = \frac{10}{7}$) ***A1 N2***

Note: Do not award the last two ***A1*** marks without the ***R1***.

[5 marks]