

DP Analysis & Approaches HL

Maths with Lemon

Lesson 9.3HL: Volumes of Revolution

Mathematician: _____

AHL 5.17	Guidance, clarification and syllabus links
Volumes of revolution	Area of the region enclosed by a curve and the y -axis in a given interval. Volumes of revolution about the x -axis or y -axis.

Warm Up 1

Write down an appropriate integral to find the area of each region, then evaluate the integral.

a) $f(x) = \ln x$

Find the area under

$$y = \ln x$$

from

$$x = 1$$

to

$$x = e.$$

$$A =$$

b) $f(x) = e^x$ and $g(x) = e^{-x}$

Find the area between

$$y = e^x$$

and

$$y = e^{-x}$$

for

$$0 \leq x \leq 1.$$

$$A =$$

Warm Up 2

Find the area enclosed by

$$y = \ln x$$

and the lines

$$y = 0$$

and

$$y = 1.$$

Hint: Since the boundaries are given as y -values, consider writing x in terms of y .



Volumes of Revolution about the x -axis

When a curve is rotated through 360° about the x -axis, it creates a solid of revolution. Complete the formula:

$$V = \int_a^b \quad \quad \quad dx$$

Example 1

Find the volume generated when

$$y = x^3, \quad 0 \leq x \leq 2,$$

is rotated around the x -axis.

Give your answer in terms of π .

Step 1: Radius

$$r =$$

Step 2: Set up the integral

$$V =$$

Step 3: Evaluate

$$V =$$

You Try

Find the volume generated when

$$y = x^2 + 1, \quad 0 \leq x \leq 3,$$

is rotated around the x -axis.

Give your answer in terms of π .

Volumes of Revolution about the y -axis

Complete the formula for rotation about the y -axis:

$$V =$$

What variable should the integral normally be with respect to?

Example 2

Find the volume generated when

$$y = x^2 - 1, \quad 1 \leq x \leq 5,$$

is rotated around the y -axis.

Give your answer in terms of π .

Step 1: Rearrange in terms of y .

$$x^2 =$$

Step 2: Convert the limits.

When $x = 1$,

$$y =$$

When $x = 5$,

$$y =$$

Step 3: Set up the integral.

$$V =$$

Step 4: Evaluate.

You Try

The curve

$$y = x^2 + 3, \quad 0 \leq x \leq 2,$$

is rotated around the y -axis.

Find the exact volume generated.

Revolving Around Both Axes

Let

$$f(x) = \frac{2x + 4}{x + 1}, \quad 0 \leq x \leq 4.$$

a) Rotation about the x -axis

Write down an appropriate integral for the volume.

$$V =$$

Evaluate the volume.

b) Rotation about the y -axis

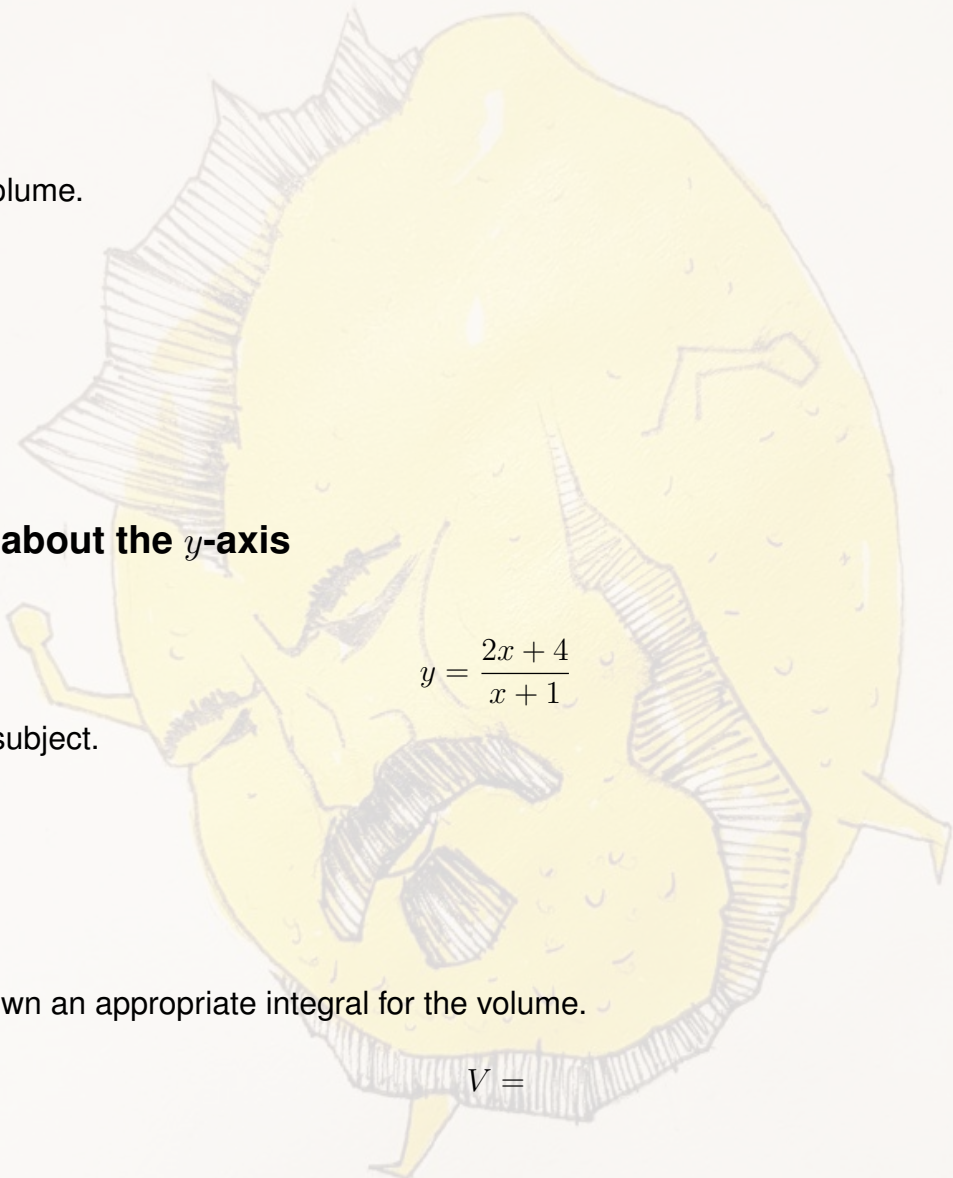
First rearrange

$$y = \frac{2x + 4}{x + 1}$$

to make x the subject.

Hence write down an appropriate integral for the volume.

$$V =$$



Solids Created from Multiple Curves

A region is bounded by the x -axis and the curves

$$y = \sin x$$

and

$$y = \cos x.$$

1. Find the point where the curves intersect.

Solve

$$\sin x = \cos x.$$

2. Rotation around the x -axis

Explain why the volume may need to be split into two integrals.

Write an appropriate exact integral:

$$V =$$

Evaluate the exact volume.

3. Rotation around the y -axis

Explain what must change in the setup when the same region is rotated around the y -axis.



Determining Bounded Area

The curves

$$y = x^2$$

and

$$y = \sqrt{x}$$

enclose a finite region.

a) Find their points of intersection.

$$x^2 = \sqrt{x}$$

b) Identify the outer and inner radii

When the region is rotated about the x -axis:

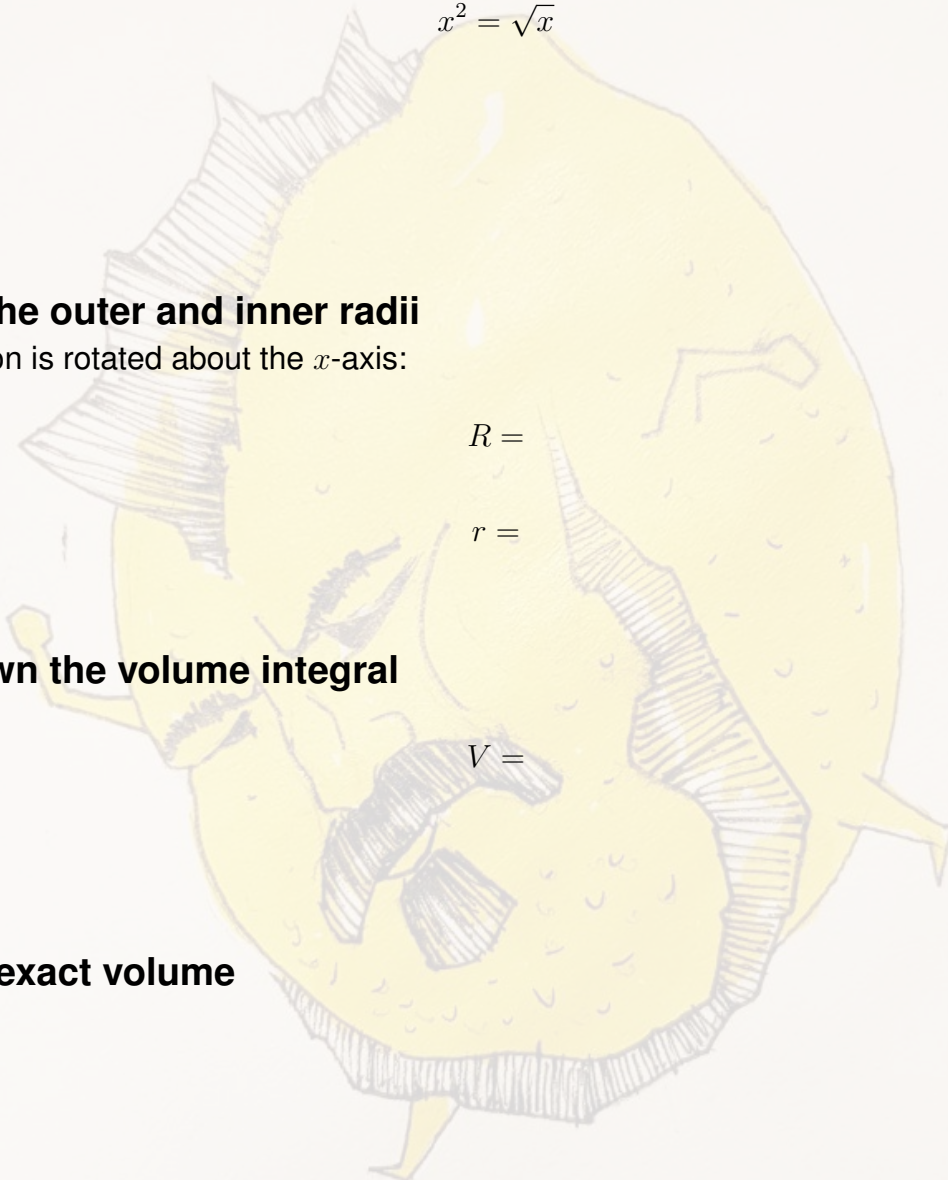
$$R =$$

$$r =$$

c) Write down the volume integral

$$V =$$

d) Find the exact volume



Rotating About a Non-Axis Vertical Line

Find the exact volume generated when the region bounded by

$$x = 1, \quad y = 1,$$

and

$$y = \ln x, \quad 1 \leq x \leq e,$$

is rotated around the line

$$x = -2.$$

Step 1: Sketch and identify the region

Step 2: Identify the radii

Outer radius:

$$R =$$

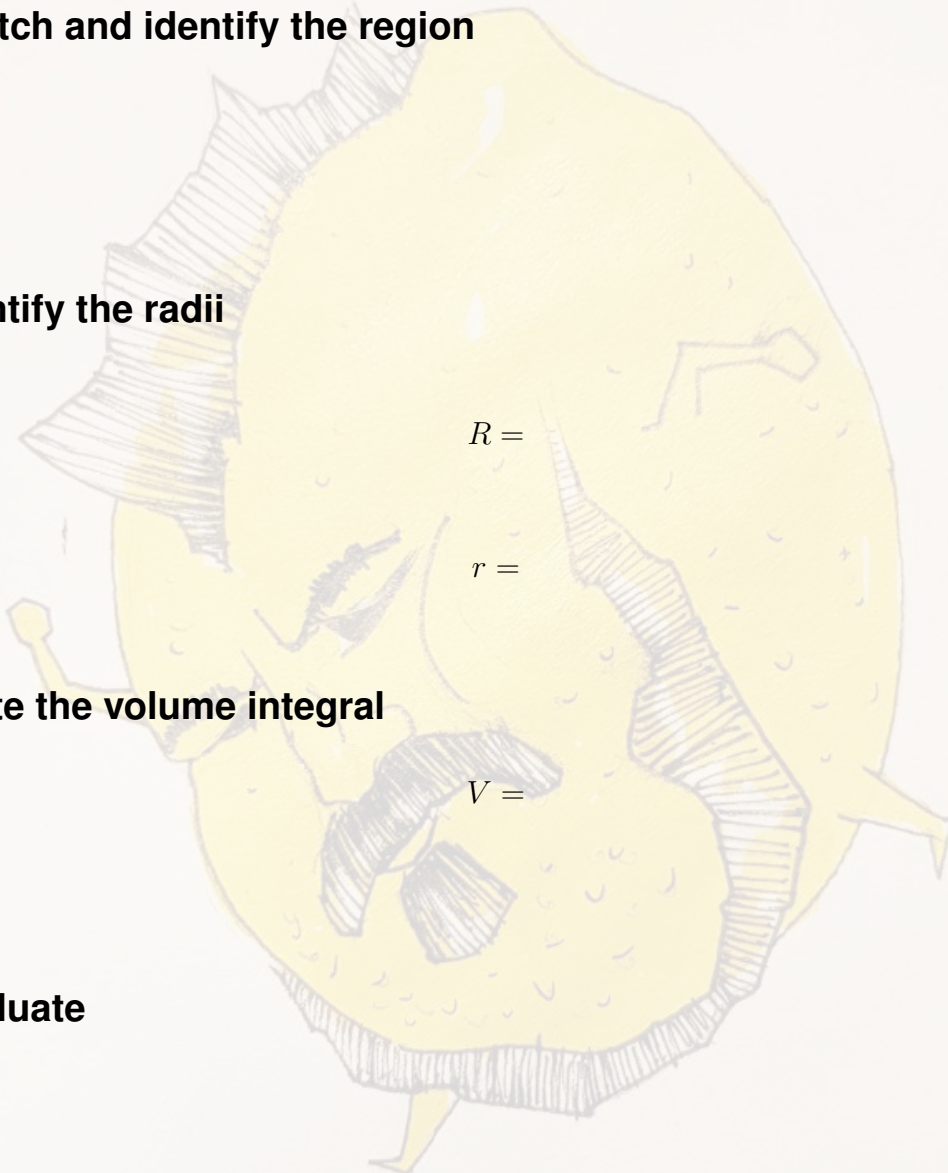
Inner radius:

$$r =$$

Step 3: Write the volume integral

$$V =$$

Step 4: Evaluate



Rotating About a Non-Axis Horizontal Line

Consider the curve

$$y = \cos x.$$

A region involving this curve is rotated through

$$2\pi$$

radians around

$$y = -1.$$

1. What does 2π radians mean?

2. Write the distance from the curve to $y = -1$.

$$R =$$

3. Hence write an appropriate volume integral.

$$V =$$

4. Expand

$$(\cos x + 1)^2 =$$

Use

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

to evaluate the integral exactly.

Exam-Style Practice

Question 1

The region bounded by

$$y = 4 - x^2,$$

the x -axis, and the y -axis is rotated through 2π radians about the x -axis.

Find the exact volume of the solid formed.

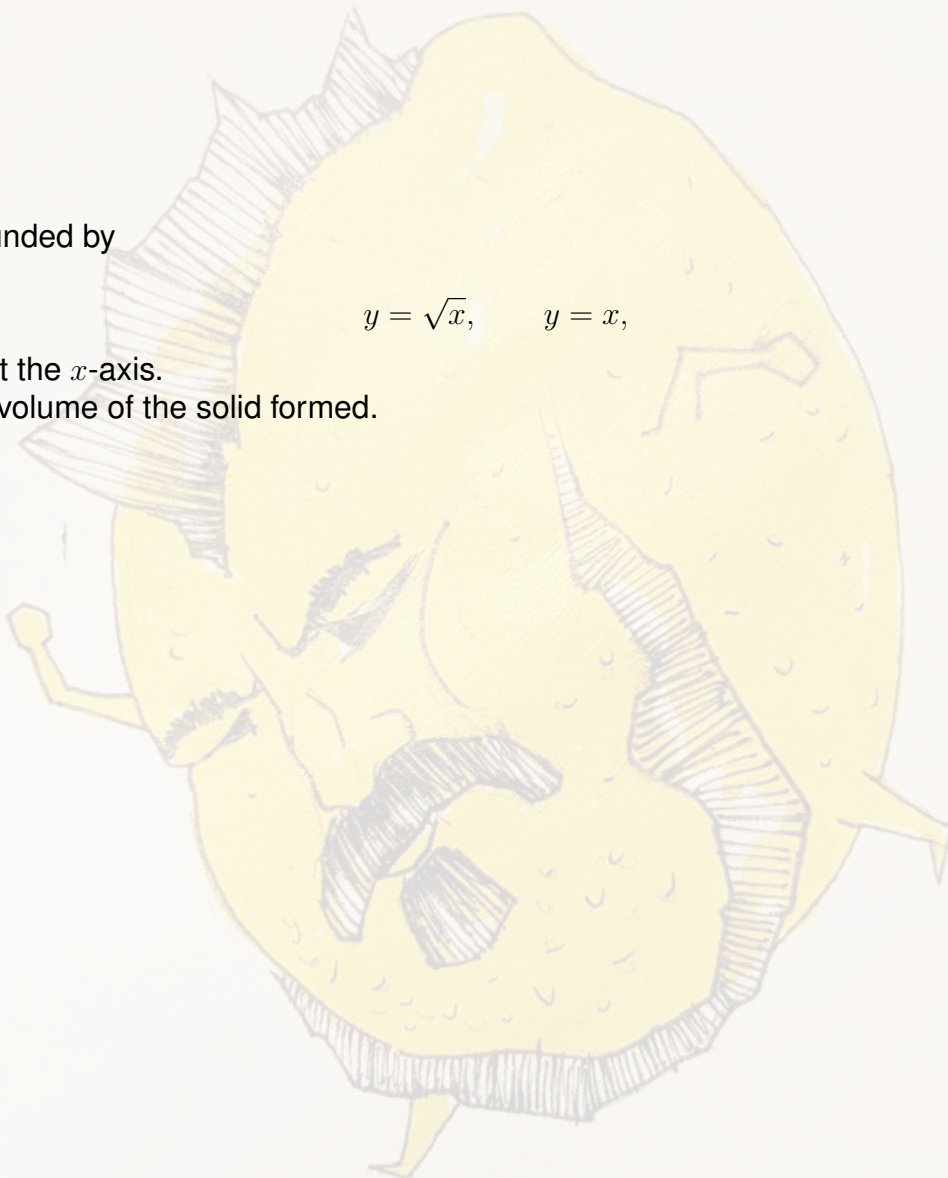
Question 2

The region bounded by

$$y = \sqrt{x}, \quad y = x,$$

is rotated about the x -axis.

Find the exact volume of the solid formed.



Question 3

The region under

$$y = e^x, \quad 0 \leq x \leq 1,$$

is rotated around the line

$$y = -2.$$

Write down an appropriate integral for the volume of the resulting solid.

Question 4

The curve

$$y = x^2$$

and the line

$$y = 4$$

bound a region.

The region is rotated around the y -axis.

(a) Find the points where the curve and line intersect.

(b) Write x^2 in terms of y .

(c) Hence find the exact volume generated.

Quick Formula Check

Complete the following:

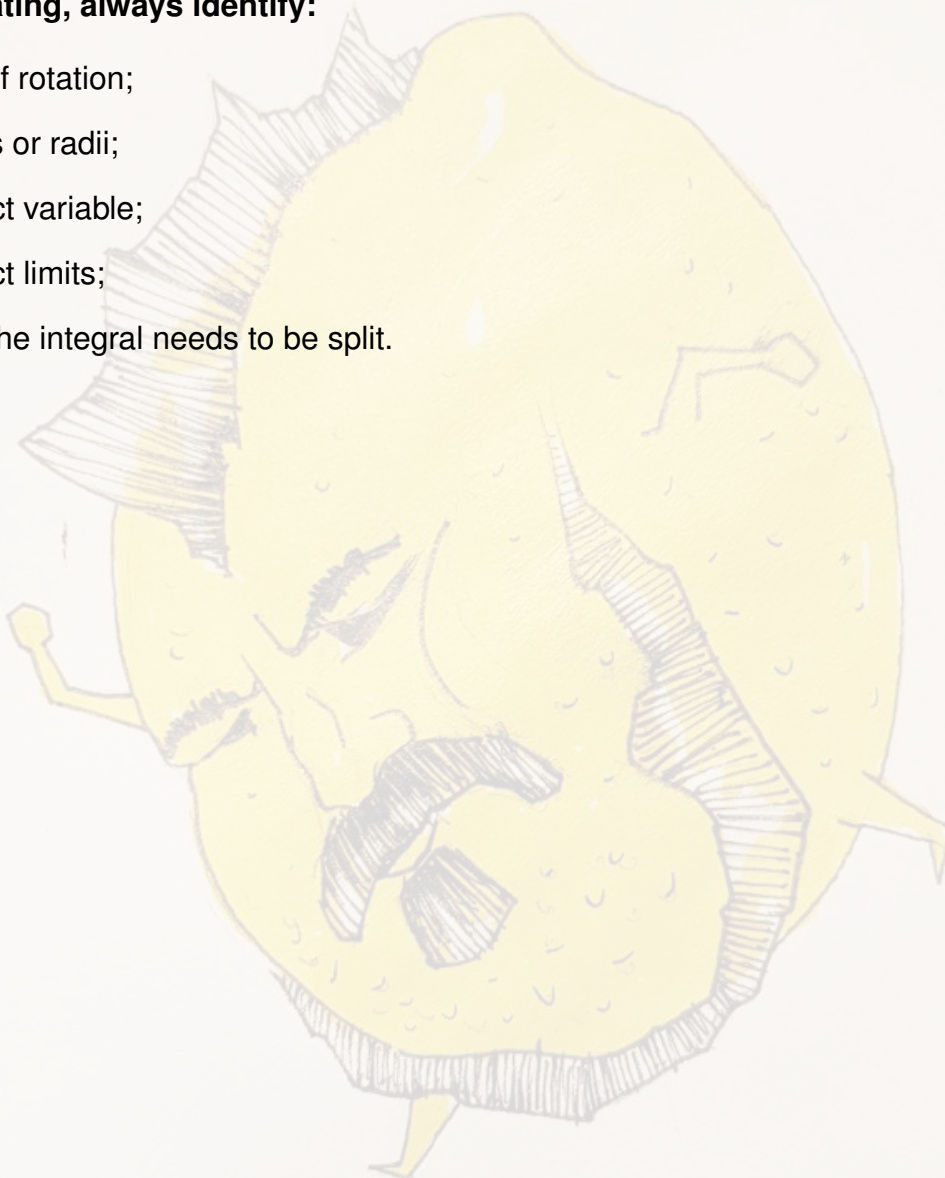
About the x -axis: $V =$

About the y -axis: $V =$

Washer method: $V =$

Before integrating, always identify:

1. the axis of rotation;
2. the radius or radii;
3. the correct variable;
4. the correct limits;
5. whether the integral needs to be split.





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Mathematics made clear.